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Pushforwards of pluricanonical bundles under morphisms to abelian varieties. (English)

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Let X be a smooth complex projective variety and let $f: X \rightarrow A$ be a morphism to an abelian variety. By the work of many people (see the introduction of the paper for a detailed account) the sheaf $f_*\omega_X$ has many important properties:

- its cohomological support loci are finite unions of translations of abelian subvarieties of $\text{Pic}^0(A)$;
- it is a GV sheaf;
- it has a canonical decomposition into pullbacks of M-regular coherent sheaves from quotients of A twisted by torsion line bundles
- its Fourier-Mukai transform is locally analytically quasi-isomorphic to a linear complex.

The purpose of this very interesting paper is to show that the same properties hold for the sheaves $f_*\omega_X^{\otimes m}$ for all integers $m \geq 2$. (This is not unexpected since, as a general rule, the pluricanonical sheaves are better behaved than the canonical sheaf).

The results are obtained as a consequence of the above properties of $f_*\omega_X$ and of the following

Theorem A. Fix $m \geq 2$; there exists a generically finite morphism $g: Y \rightarrow X$ such that $f_*\omega_X^{\otimes m}$ is a direct summand of $(f \circ g)_*\omega_Y$.

Theorem A is in turn a consequence of

Theorem B. There is an isogeny $\phi: A' \rightarrow A$ such that $f'_*\omega_{X'}^{\otimes m}$ is globally generated, where $f': X' \rightarrow A'$ is obtained from $f: X \rightarrow A$ by taking base change with ϕ .

The authors also prove a more refined version of the decomposition into pullbacks of M-regular coherent sheaves from quotients of A twisted by torsion line bundles in the case when $m \geq 2$ and f is the Albanese morphism.

Some results on the pluricanonical linear series of normal varieties of maximal Albanese dimension with at most canonical singularities are obtained as an application of the previous results.

Reviewer: [Rita Pardini \(Pisa\)](#)

MSC:

14F06 Sheaves in algebraic geometry

14K05 Algebraic theory of abelian varieties

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direct images; pluricanonical bundles; abelian varieties; non-vanishing loci; singular Hermitian metrics; pluricanonical systems

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