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Strongly minimal groups in o-minimal structures. (English) Zbl 1468.14102

B. Zilber’s trichotomy conjecture introduced in [Sibirsk. Mat. Zh. 25, 71–88 (1984; Zbl 0581.03022)] was disproved by E. Hrushovski [J. Amer. Math. Soc. 62, 147–166 (1993; Zbl 0804.03020)]. However, this conjecture is true in various restricted settings. This paper proves Zilber’s trichotomy conjecture for strongly minimal expansion of 2-dimensional groups, definable in o-minimal structures. The main theorem is as follows:

Let $\mathcal{M}$ be an o-minimal expansion of a real closed field, $(G; +)$ be a 2-dimensional group definable in $\mathcal{M}$, and $\mathcal{D} = (G; +, \ldots)$ be a strongly minimal structure, all of whose atomic relations definable in $\mathcal{M}$. If $\mathcal{D}$ is not locally modular, then an algebraic closed field $K$ is interpretable in $\mathcal{D}$, and the group $G$, with all its induced $\mathcal{D}$-structure, is definably isomorphic to an algebraic $K$-group with all its induced $K$-structure.

It is a generalization of [A. Hasson et al., Proc. London Math. Soc. (3) 97, 117–154 (2008; Zbl 1153.03011)] which treats the case in which $G$ is the algebraic closure $K = R[i]$ and $\mathcal{D}$ is a structure generated by an $\mathcal{M}$-definable function, and its proof follows the same strategy as the Hasson’s paper; that is, constructing a field configuration and using Hrushovski’s result that a strongly minimal structure admitting a field construction interprets an algebraically closed field.

A $\mathcal{D}$-definable subset of $G^2$ whose Morley rank is one is called a plane curve in this paper. The paper establishes the necessary ingredients for the proof of the main theorem in several distinct steps. In each step, resemblances of $\mathcal{D}$-definable sets to complex algebraic sets are demonstrated including finiteness of the frontiers of plane curves, finiteness of their poles and their intersection theory. The algebraically closed field $K$ is defined as the collection of all Jacobian matrices at zero of local smooth maps from $G$ to $G$ whose graph is contained in a plane curve by identifying them with the matrices in $M_2(R)$.

Reviewer: Fujita Masato (Kure)

MSC:
14P25 Topology of real algebraic varieties
03C64 Model theory of ordered structures; o-minimality
03C45 Classification theory, stability, and related concepts in model theory

Keywords:
o-minimality; strongly minimal groups; Zilber’s conjecture

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References:


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