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On feebly compact paratopological groups. (English) Zbl 1468.22003

Summary: We obtain many results and solve some problems about feebly compact paratopological groups. We obtain necessary and sufficient conditions for such a group to be topological. One of them is the quasiregularity. We prove that each 2-pseudocompact paratopological group is feebly compact and that each Hausdorff σ-compact feebly compact paratopological group is a compact topological group. Our particular attention concerns periodic and topologically periodic groups. We construct examples of various compact-like paratopological groups which are not topological groups, among them a $T_0$ sequentially compact group, a $T_1$ 2-pseudocompact group, a functionally Hausdorff countably compact group (under the axiomatic assumption that there is an infinite torsion-free Abelian countably compact topological group without non-trivial convergent sequences), and a functionally Hausdorff second countable group sequentially pracomact group. We prove that the product of a family of feebly compact paratopological groups is feebly compact, and that a paratopological group $G$ is feebly compact provided it has a feebly compact normal subgroup $H$ such that a quotient group $G/H$ is feebly compact. For our research we also study some general constructions of paratopological groups. We extend the well-known construction of Raikov completion of a $T_0$ topological group to the class of paratopological groups. We investigate cone topologies of paratopological groups which provide a general tool for constructing pathological examples, especially examples of compact-like paratopological groups with discontinuous inversion. We find a simple interplay between the algebraic properties of a basic cone subsemigroup $S$ of a group $G$ and compact-like properties of two basic semigroup topologies generated by $S$ on the group $G$.

MSC:

22A15 Structure of topological semigroups
54D10 Lower separation axioms ($T_0$–$T_3$, etc.)
54H11 Topological groups (topological aspects)
54H99 Connections of general topology with other structures, applications

Keywords:
paratopological group; continuity of the inverse; totally countably compact paratopological group; countably compact paratopological group; 2-pseudocompact paratopological group; saturated paratopological group; topologically periodic paratopological group; product of paratopological groups; pseudocompact topological group; countably compact topological group; countably pracomact space

Full Text: DOI arXiv

References:

[11] Banakh, T.; Dimitrova, S.; Gutik, O., Embedding the bicyclic semigroup into countably compact topological semigroups,
[90] Tkachenko, M., Countably compact and pseudocompact topologies on free Abelian groups, Soviet Math. (Iz. VUZ), 34, 5, 79-86 (1990) · Zbl 0714.22001
[92] Tkachenko, M., Productive properties in topological groups, preprint (version April 18, 2013)

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