Allouche, J.-P.
The zeta-regularized product of odious numbers. (English) [Zbl 1469.11348]

In the paper, the zeta-regularized product \( \prod_{i=1}^{\infty} \lambda_i := e^{-\zeta'_{\Lambda}(0)} \) is studied (here \((\lambda_i)_{i \geq 1}\) is a sequence of positive numbers, and \( \zeta_{\Lambda}(s) := \sum_{i \geq 1} (1/\lambda_i^s) \) is the Dirichlet series convergent for enough large real part of \(s\)).

By \( \mathcal{O} \) and \( \mathcal{E} \) denote the sets of odious positive integers (integers whose sum of binary digits is odd) and the set of evil numbers (integers whose sum of binary digits is even), respectively. Let \( Q := \prod_{n \geq 1} \left( \left(2^n/(2n+1)\right)^{\varepsilon_n} \right) \) for \( \varepsilon_n \) equal to \(+1\) (for even such sum) or \(-1\) (when such sum is odd). Then it is proved that

\[
\prod_{n \in \mathcal{O}} n = (2\pi)^{1/4}Q^{-1/2} \quad \text{and} \quad \prod_{n \in \mathcal{E}} n = (2\pi)^{1/4}Q^{1/2}.
\]

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MSC:
11M41 Other Dirichlet series and zeta functions
40A20 Convergence and divergence of infinite products
11A63 Radix representation; digital problems
11B85 Automata sequences

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References:


[21] Smirnov, V. V., The \( \zeta \)-regularized product over all primes (2015), preprint, available at


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