The idea of generating a proper orthogonal matrix \( A \) in Euclidean space via a skew-symmetric matrix \( S \) can also be applied in Lorentzian or Minkowski spaces, i.e. a Euclidean space equipped with a Lorentzian inner product. This leads to a so called proper semi-orthogonal matrix \( B \), proper in both cases denote that the determinant is equal to 1. A transformation \( f: A \mapsto B \) is defined in this paper between such matrices generated from the same skew-symmetric matrix.

As proper orthogonal and semi-orthogonal matrices generate rotations in the respective spaces the entries of the used skew symmetric matrix are related to their rotation axes. The related vector \( s \) carrying the entries of \( S \) is kept invariant with respect to both transformations generated by \( A \) and \( B \).

Generating \( A \) and \( B \) via a one parametric matrix \( S \) yields rotations which keep the cone with base curve \( s \) and apex in the origin invariant. Example one shows this behavior.

Using a two-parametric vector \( s \), i.e. a parametric representation of a surface, to generate the skew-symmetric matrix \( S \) one achieves two-parametric proper orthogonal and semi-orthogonal matrices. Applying these Lie group actions on a curve contained on the given surface generates orbit surfaces, which share exactly this curve either in Euclidean or in Minkowski space. An example with figures is given in both spaces.

This theory is then further extended to the respective dual spaces with similar results in case of starting with a dual curve \( s \), what is shown in a third example.

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