
Summary: Consider an elliptic parameter $k$; we introduce a family of $Z^u$-Dirac operators $(K(u))_{u \in C}$, relate them to the $Z$-massive Laplacian of C. Boutillier et al. [Invent. Math. 208, No. 1, 109–189 (2017; Zbl 1372.82016)], and extend to the full $Z$-invariant case the results of R. Kenyon [Invent. Math. 150, No. 2, 409–439 (2002; Zbl 1038.58037)] on discrete holomorphic and harmonic functions, which correspond to the case $k = 0$. We prove through combinatorial identities, how and why the $Z^u$-Dirac and $Z$-massive Laplacian operators appear in the $Z$-invariant Ising model, considering the case of infinite and finite isoradial graphs. More precisely, consider the dimer model on the Fisher graph $G^F$ arising from a $Z$-invariant Ising model. We express coefficients of the inverse Fisher Kasteleyn operator as a function of the inverse $Z^u$-Dirac operator and also as a function of the $Z$-massive Green function; in particular this proves a (massive) random walk representation of important observables of the Ising model. We prove that the squared partition function of the Ising model is equal, up to a constant, to the determinant of the $Z$-massive Laplacian operator with specific boundary conditions, the latter being the partition function of rooted spanning forests. To show these results, we relate the inverse Fisher Kasteleyn operator and that of the dimer model on the bipartite graph $G^Q$ arising from the XOR-Ising model, and we prove matrix identities between the Kasteleyn matrix of $G^Q$ and the $Z^u$-Dirac operator, that allow to reach inverse matrices as well as determinants.

MSC:
- 82B20 Lattice systems (Ising, dimer, Potts, etc.) and systems on graphs arising in equilibrium statistical mechanics
- 82B23 Exactly solvable models; Bethe ansatz
- 05A19 Combinatorial identities, bijective combinatorics
- 33E05 Elliptic functions and integrals

Keywords:
dimer model; discrete massive harmonic and holomorphic functions; Ising model; massive Laplacian and Dirac operators; spanning forests and spanning trees; $Z$-invariance

Full Text: DOI arXiv

References:


[41] P. W. Kasteleyn, "Graph theory and crystal physics," Graph Theory and Theoretical Physics, Academic Press, London,


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