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Variations on twins in permutations. (English) [Zbl 1470.05009]

Summary: Let $\pi$ be a permutation of the set $[n] = \{1, 2, \ldots, n\}$. Two disjoint order-isomorphic subsequences of $\pi$ are called twins. How long twins are contained in every permutation? The well known Erdős-Szekeres theorem implies that there is always a pair of twins of length $\Omega(\sqrt{n})$. On the other hand, by a simple probabilistic argument M. Gawron [Izomorficzne podstruktury w s lowach i permutacjach. Kraków: Uniwersytet Jagielloński (Master Thesis) (2014)] proved that for every $n \geq 1$ there exist permutations with all twins having length $O(n^{2/3})$. He conjectured that the latter bound is the correct size of the longest twins guaranteed in every permutation. We support this conjecture by showing that almost all permutations contain twins of length $\Omega(n^{2/3}/\log n)$. Recently, B. Bukh and O. Rudenko [SIAM J. Discrete Math. 34, No. 3, 1620–1622 (2020; Zbl 1450.05001)] have tweaked our proof and removed the log-factor. For completeness, we also present our version of their proof (see Remark 2 below on the interrelation between the two proofs).

In addition, we study several variants of the problem with diverse restrictions imposed on the twins. For instance, if we restrict attention to twins avoiding a fixed permutation $\tau$, then the corresponding extremal function equals $\Theta(\sqrt{n})$, provided that $\tau$ is not monotone. In case of block twins (each twin occupies a segment) we prove that it is $(1+o(1)) \frac{\log n}{\log \log n}$, while for random permutations it is twice as large. For twins that jointly occupy a segment (tight twins), we prove that for every $n$ there are permutations avoiding them on all segments of length greater than 24.

MSC:
05A05 Permutations, words, matrices
05A16 Asymptotic enumeration
60C05 Combinatorial probability

Keywords:
subpermutation; permutation pattern; twins

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References:


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