

Polishchuk, Alexander; Van den Bergh, Michel

Semiorthogonal decompositions of the categories of equivariant coherent sheaves for some reflection groups. (English) [Zbl 1470.14037](#)

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Summary: We consider the derived category $D_G^b(V)$ of coherent sheaves on a complex vector space V equivariant with respect to an action of a finite reflection group G . In some cases, including Weyl groups of type A, B, G_2, F_4 , as well as the groups $G(m, 1, n) = (\mu_m)^n \rtimes S_n$, we construct a semiorthogonal decomposition of this category, indexed by the conjugacy classes of G . The pieces of this decompositions are equivalent to the derived categories of coherent sheaves on the quotient-spaces $V^g/C(g)$, where $C(g)$ is the centralizer subgroup of $g \in G$. In the case of the Weyl groups the construction uses some key results about the Springer correspondence, due to Lusztig, along with some formality statement generalizing a result of *P. Deligne* [Publ. Math., Inst. Hautes Étud. Sci. 52, 137–252 (1980; [Zbl 0456.14014](#))]. We also construct global analogs of some of these semiorthogonal decompositions involving derived categories of equivariant coherent sheaves on C^n , where C is a smooth curve.

MSC:

- [14F08](#) Derived categories of sheaves, dg categories, and related constructions in algebraic geometry
- [18G80](#) Derived categories, triangulated categories
- [55N91](#) Equivariant homology and cohomology in algebraic topology

Cited in **2** Reviews
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Keywords:

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