Polishchuk, Alexander; Van den Bergh, Michel
Semiorthogonal decompositions of the categories of equivariant coherent sheaves for some reflection groups. (English) 

Summary: We consider the derived category $D^b_G(V)$ of coherent sheaves on a complex vector space $V$ equivariant with respect to an action of a finite reflection group $G$. In some cases, including Weyl groups of type $A, B, G_2, F_4$, as well as the groups $G(m,1,n) = (\mathbb{Z}/m\mathbb{Z})^n \rtimes S_n$, we construct a semiorthogonal decomposition of this category, indexed by the conjugacy classes of $G$. The pieces of these decompositions are equivalent to the derived categories of coherent sheaves on the quotient-spaces $V^g/C(g)$, where $C(g)$ is the centralizer subgroup of $g \in G$. In the case of the Weyl groups the construction uses some key results about the Springer correspondence, due to Lusztig, along with some formality statement generalizing a result of P. Deligne [Publ. Math., Inst. Hautes Étud. Sci. 52, 137–252 (1980; Zbl 0456.14014)]. We also construct global analogs of some of these semiorthogonal decompositions involving derived categories of equivariant coherent sheaves on $C^n$, where $C$ is a smooth curve.

MSC:
14F08 Derived categories of sheaves, dg categories, and related constructions in algebraic geometry
18G80 Derived categories, triangulated categories
55N91 Equivariant homology and cohomology in algebraic topology

Keywords:
derived category; semiorthogonal decomposition; equivariant sheaf; Springer correspondence; reflection group; Hochschild homology; equivariant cohomology

Full Text: DOI arXiv

References: