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Two notes on the O’Hara energies. (English) [Zbl 1470.57015]

Summary: The O’Hara energies, introduced by J. O’Hara in [Topology 30, No. 2, 241–247 (1991; Zbl 0733.57005)], were proposed to answer the question of what is a “good” figure in a given knot class. A property of the O’Hara energies is that the “better” the figure of a knot is, the less the energy value is. In this article, we discuss two topics on the O’Hara energies. First, we slightly generalize the O’Hara energies and consider a characterization of its finiteness. The finiteness of the O’Hara energies was considered by S. Blatt in [J. Knot Theory Ramifications 21, No. 1, Article ID 1250010, 9 p. (2012; Zbl 1238.57007)] who used the Sobolev-Slobodeckij space, and naturally we consider a generalization of this space. Another fundamental problem is to understand the minimizers of the O’Hara energies. This problem has been addressed in several papers, some of them based on numerical computations. In this direction, we discuss a discretization of the O’Hara energies and give some examples of numerical computations. Particular one of the O’Hara energies, called the Möbius energy thanks to its Möbius invariance, was considered by D. Kim and R. Kusner in [Exp. Math. 2, No. 1, 1–9 (1993; Zbl 0818.57007)], and S. Scholtes in [J. Knot Theory Ramifications 23, No. 9, Article ID 1450045, 16 p. (2014; Zbl 1312.49054)] established convergence properties. We apply their argument in general since the argument does not rely on Möbius invariance.

MSC:
57K10 Knot theory
53A04 Curves in Euclidean and related spaces
49Q10 Optimization of shapes other than minimal surfaces
46E35 Sobolev spaces and other spaces of “smooth” functions, embedding theorems, trace theorems
49M25 Discrete approximations in optimal control

Keywords: knot energy; Sobolev-Slobodeckij space; discrete approximation; minimizer; numerical computation

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References:


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