Suppose $G$ is a simply connected, simple compact Lie group with trivial $\pi_4(G)$ and $M$ is an oriented closed compact 5-manifold with $\pi_1(M) \cong \mathbb{Z}/c\mathbb{Z}$ and $H_2(M)$ being torsion free. The set $[M, BG]$ of pointed homotopy classes of maps from $M$ to $BG$ is $\mathbb{Z}/c\mathbb{Z}$. Hence the isomorphism class of a principal $G$-bundle $P$ over $M$ is determined by a mod-$c$ congruence class $k \in \mathbb{Z}/c\mathbb{Z}$. The gauge group $\mathcal{G}_k(M)$ is the topological group consisting of $G$-equivariant automorphisms of $P$ that fix $M$.

In this paper the author studies the homotopy types of looped gauge groups for 5-manifolds. In particular, he shows that $\Omega^3\mathcal{G}_k(M)$ decomposes as a product of double looped gauge groups for Moore spaces and some looped mapping spaces of $G$ when $6 \nmid c$, and so does $\Omega^3\mathcal{G}_k(M)$ when $2 \nmid c$, $M$ is stably parallelizable and has one 5-dimensional cell.

In Section 2 the author proves the key proposition that leads to his main theorems: If $M$ has a subcomplex $Y$ such that $[Y, BG] = 0$ and the suspension of the inclusion $\Sigma^{i+1}Y \hookrightarrow \Sigma^{i+1}M$ has a homotopy left inverse, then $\Omega^i\mathcal{G}_k(M) \cong \Omega^i\mathcal{G}_k(M/Y) \times \Omega^i\text{Map}^*(Y, G)$. Then in Sections 3-5 he calculates the homotopy types of $\Sigma^i M$ and decomposes $\Omega^i\mathcal{G}_k(M)$ into products of looped gauge groups for Moore spaces and some looped mapping spaces of $G$ when $2 \nmid c$ and when $6 \nmid c$, $M$ is stably parallelizable and has one 5-cell. Furthermore he studies the homotopy types of gauge groups for Moore spaces in Section 6.

In Sections 7 and 8 the author gives two applications of the main theorems. The first application is to calculate the $p^{\text{th}}$ homotopy exponent of $\mathcal{G}_k(M)$, which is the least power of $p$ that annihilates all $p$-torsions in $\pi_*(\mathcal{G}_k(M))$. Using the decomposition of $\mathcal{G}_k(M)$ and the knowledge of homotopy exponents of spheres, Moore spaces and Lie groups he gives upper bounds on the homotopy exponents of $\mathcal{G}_k(M)$ for different $G$’s. The second application is to compute the localized homotopy groups $\pi_*(\mathcal{G}_k^C(M))$ for $G = SU, Spin$. He shows that they have periodicity similar to the Bott periodicity of $\pi_*(SU)$ and $\pi_*(Spin)$.

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MSC:

57S05 Topological properties of groups of homeomorphisms or diffeomorphisms
55P15 Classification of homotopy type
55P40 Suspensions
54C35 Function spaces in general topology
58B05 Homotopy and topological questions for infinite-dimensional manifolds
81T13 Yang-Mills and other gauge theories in quantum field theory

Keywords:

gauge group; 5-manifolds; homotopy decompositions; homotopy exponents; homotopy groups

Full Text: DOI arXiv

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Huang, Ruizhi

Homotopy of gauge groups over non-simply-connected five-dimensional manifolds. (English)

Zbl 1470.57051

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