On the beta prime prior for scale parameters in high-dimensional Bayesian regression models. (English) Zbl 1470.62103

Summary: We study a high-dimensional Bayesian linear regression model in which the scale parameter follows a general beta prime distribution. Under the assumption of sparsity, we show that an appropriate selection of the hyperparameters in the beta prime prior leads to the (near) minimax posterior contraction rate when \( p \gg n \). For finite samples, we propose a data-adaptive method for estimating the hyperparameters based on the marginal maximum likelihood (MML). This enables our prior to adapt to both sparse and dense settings and, under our proposed empirical Bayes procedure, the MML estimates are never at risk of collapsing to zero. We derive an efficient Monte Carlo expectation-maximization (EM) and variational EM algorithm for our model, which are available in the R package NormalBetaPrime. Simulations and an analysis of a gene expression data set illustrate our model’s self-adaptivity to varying levels of sparsity and signal strengths.

MSC:
62J05 Linear regression; mixed models
62F15 Bayesian inference
62P10 Applications of statistics to biology and medical sciences; meta analysis

Keywords:
beta prime density; empirical Bayes; high-dimensional data; posterior contraction; scale mixtures of normal distributions

Software:
glmnet; NormalBetaPrime; EBayesThresh

Full Text: DOI

References:


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