

**Goodearl, Kenneth R.; Yakimov, M. T.**

**The Berenstein-Zelevinsky quantum cluster algebra conjecture.** (English) Zbl 1471.13046  
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Summary: We prove the Berenstein-Zelevinsky conjecture that the quantized coordinate rings of the double Bruhat cells of all finite-dimensional connected, simply connected simple algebraic groups admit quantum cluster algebra structures with initial seeds as specified by *A. Berenstein* and *A. Zelevinsky* [*Adv. Math.* 195, No. 2, 405–455 (2005; [Zbl 1124.20028](#))]. We furthermore prove that the corresponding upper quantum cluster algebras coincide with the constructed quantum cluster algebras and exhibit a large number of explicit quantum seeds. Along the way a detailed study of the properties of quantum double Bruhat cells from the viewpoint of noncommutative UFDs is carried out and a quantum analog of the Fomin-Zelevinsky twist map is constructed and investigated for all double Bruhat cells. The results are valid over base fields of arbitrary characteristic and the deformation parameter is only assumed to be a non-root-of-unity.

**MSC:**

[13F60](#) Cluster algebras

[20G42](#) Quantum groups (quantized function algebras) and their representations

[16T20](#) Ring-theoretic aspects of quantum groups

[17B37](#) Quantum groups (quantized enveloping algebras) and related deformations

[14M15](#) Grassmannians, Schubert varieties, flag manifolds

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**Keywords:**

quantum cluster algebras; quantum double Bruhat cells; quantum nilpotent algebras

**Full Text:** [DOI](#) [arXiv](#)

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