Mazanti, Guilherme; **Boussaada, Islam**; **Niculescu, Silviu-Iulian**

Multiplicity-induced-dominancy for delay-differential equations of retarded type. (English)

Zbl 1471.34140
J. Differ. Equations 286, 84-118 (2021).

Consider a delay equation of the form

$$y^{(n)}(t) + \sum_{k=0}^{n-1} a_k y^{(k)}(t) + \sum_{k=0}^{n-1} \alpha_k y^{(k)}(t-\tau) = 0$$

with real coefficients and $\tau > 0$. The related characteristic function is defined as

$$\Delta(s) = s^n + \sum_{k=0}^{n-1} a_k + e^{-s\tau} \sum_{k=0}^{n-1} \alpha_k s^k.$$

Let $s_0 \in \mathbb{R}$. It is obtained a criterion for s_0 to be a root of multiplicity $2n$ of $\Delta(s)$. The conditions for s_0 to be a strictly dominant root are proved.

If s_0 is a root of multiplicity $2n$ of $\Delta(s)$, then the trivial solution is exponentially stable if $a_{n-1} > -n^2/\tau$.

Some applications for linear control systems are given.

Reviewer: Nikita V. Artamonov (Moskva)

MSC:

34K20 Stability theory of functional-differential equations
34K06 Linear functional-differential equations
34K35 Control problems for functional-differential equations
93D15 Stabilization of systems by feedback
33C90 Applications of hypergeometric functions

Keywords:

time-delay functional-differential equation, stability, confluent hypergeometric function

Software:

DLMF; p3delta; Python

Full Text: DOI arXiv

References:

[1] Armstrong, E. S.; Tripp, J. S., An application of multivariable design techniques to the control of the National Transonic Facility (August 1981), NASA, Technical Paper 1887

[2] Avellar, C. E.; Hale, J. K., On the zeros of exponential polynomials, J. Math. Anal. Appl., 73, 2, 434-452 (1980) · Zbl 0435.30005

[3] Barnett, S., Polynomials and Linear Control Systems, Monographs and Textbooks in Pure and Applied Mathematics, vol. 77 (1983), Marcel Dekker, Inc.: Marcel Dekker, Inc. New York · Zbl 0528.93003

[4] Bellman, R.; Cooke, K. L., Differential-Difference Equations (1963), Academic Press: Academic Press New York-London · Zbl 0105.06402

[5] Berenstein, C. A.; Gay, R., Complex Analysis and Special Topics in Harmonic Analysis (1995), Springer-Verlag: Springer-Verlag New York · Zbl 0837.30001

[6] Blondel, V. D.; Gürbüzbalaban, M.; Megretski, A.; Overton, M. L., Explicit solutions for root optimization of a polynomial family with one affine constraint, IEEE Trans. Autom. Control, 57, 12, 3078-3089 (2012) · Zbl 1369.93220

[7] Boussaada, I.; Mazanti, G.; Niculescu, S.-I.; Huynh, J.; Sim, F.; Thomas, M., Partial pole placement via delay action: a Python software for delayed feedback stabilizing design, (2020 24th International Conference on System Theory, Control and

Computing (ICSTCC) (2020)), 196-201

[8] Boussaada, I.; Niculescu, S.-I., Characterizing the codimension of zero singularities for time-delay systems: a link with Vandermonde and Birkhoff incidence matrices, Acta Appl. Math., 145, 47-88 (2016) · Zbl 1383.34093

[9] Boussaada, I.; Niculescu, S.-I., Tracking the algebraic multiplicity of crossing imaginary roots for generic quasipolynomials: a Vandermonde-based approach, IEEE Trans. Autom. Control, 61, 6, 1601-1606 (2016) · Zbl 1359.93387

[10] Boussaada, I.; Niculescu, S.-I.; El Ati, A.; Perez-Ramos, R.; Trabelsi, K. L., Multiplicity-induced-dominancy in parametric second-order delay differential equations: analysis and application in control design, ESAIM Control Optim. Calc. Var., 26 (2020) · Zbl 1453.34098

[11] Boussaada, I.; Tliba, S.; Niculescu, S.-I.; Ünal, H. U.; Vyhlídal, T., Further remarks on the effect of multiple spectral values on the dynamics of time-delay systems. Application to the control of a mechanical system, Linear Algebra Appl., 542, 589-604 (2018) · Zbl 1420.34089

[12] Boussaada, I.; Ünal, H. U.; Niculescu, S.-I., Multiplicity and stable varieties of time-delay systems: a missing link, (Proceedings of the 22nd International Symposium on Mathematical Theory of Networks and Systems (MTNS). Proceedings of the 22nd International Symposium on Mathematical Theory of Networks and Systems (MTNS), Minneapolis, MN, USA (July 2016)), 188-194

[13] Buchholz, H., The Confluent Hypergeometric Function with Special Emphasis on Its Applications, Springer Tracts in Natural Philosophy, vol. 15 (1969), Springer-Verlag New York Inc.: Springer-Verlag New York Inc. New York, Translated from the German by H. Lichtblau and K. Wetzel · Zbl 0169.08501

[14] Chen, R., Output feedback stabilization of linear systems (June 1979), University of Florida, PhD thesis

[15] Cooke, K. L.; van den Driessche, P., On zeroes of some transcendental equations, Funkc. Ekvacioj, 29, 1, 77-90 (1986) · Zbl 0603.34069

[16] Cruz, M. A.; Hale, J. K., Stability of functional differential equations of neutral type, J. Differ. Equ., 7, 334-355 (1970) · Zbl 0191.38901

[17] Datko, R., Linear autonomous neutral differential equations in a Banach space, J. Differ. Equ., 25, 2, 258-274 (1977) · Zbl 0402.34066

[18] Diekmann, O.; van Gils, S. A.; Verduyn Lunel, S. M.; Walther, H.-O., Delay Equations: Functional-, Complex-, and Nonlinear Analysis, Applied Mathematical Sciences, vol. 110 (1995), Springer-Verlag: Springer-Verlag New York · Zbl 0826.34002

[19] Engelborghs, K.; Roose, D., On stability of LMS methods and characteristic roots of delay differential equations, SIAM J. Numer. Anal., 40, 2, 629-650 (2002) · Zbl 1021.65040

[20] Erdélyi, A.; Magnus, W.; Oberhettinger, F.; Tricomi, F. G., Higher Transcendental Functions. Vol. I (1981), Robert E. Krieger Publishing Co., Inc.: Robert E. Krieger Publishing Co., Inc. Melbourne, Fla, Based on notes left by Harry Bateman, With a preface by Mina Rees, With a foreword by E.C. Watson, Reprint of the 1953 original · Zbl 0051.30303

[21] Gonzalez-Vega, L., Applying quantifier elimination to the Birkhoff interpolation problem, J. Symb. Comput., 22, 1, 83-103 (1996) · Zbl 0870.65007

[22] Gromova, P. S., The asymptotic behavior of the roots of quasipolynomials which are large in absolute value and Lie near the imaginary axis, Tr. Sem. Teor. Diff. Urav. Otklon. Argumentom Univ. Druž. Nar. Patrisa Lumumby, 6, 109-124 (1968) · Zbl 0221.30002

[23] Gu, K.; Kharitonov, V. L.; Chen, J., Stability of Time-Delay Systems. Control Engineering (2003), Birkhäuser Boston, Inc.: Birkhäuser Boston, Inc. Boston, MA

[24] Ha, T. T.; Gibson, J. A., A note on the determinant of a functional confluent Vandermonde matrix and controllability, Linear Algebra Appl., 30, 69-75 (1980) · Zbl 0445.15009

[25] Hale, J. K.; Verduyn Lunel, S. M., Introduction to Functional Differential Equations, Applied Mathematical Sciences., vol. 99 (1993), Springer-Verlag: Springer-Verlag New York · Zbl 0787.34002

[26] Hale, J. K.; Verduyn Lunel, S. M., Stability and control of feedback systems with time delays, Time Delay Systems: Theory and Control. Time Delay Systems: Theory and Control, Int. J. Syst. Sci., 34, 8-9, 497-504 (2003) · Zbl 1052.93028

[27] Henry, D., Linear autonomous neutral functional differential equations, J. Differ. Equ., 15, 106-128 (1974) · Zbl 0294.34047

[28] Kailath, T., Linear Systems, Prentice-Hall Information and System Sciences Series (1980), Prentice-Hall, Inc.: Prentice-Hall, Inc. Englewood Cliffs, N.J. · Zbl 0458.93025

[29] Kharitonov, V. L., A stability criterion for a family of quasipolynomials of delaying type, Avtom. Telemeh., 52, 2, 73-82 (1991)

[30] Krasovskiĭ, N. N., Stability of Motion. Applications of Lyapunov's Second Method to Differential Systems and Equations with Delay (1963), Stanford University Press: Stanford University Press Stanford, Calif. · Zbl 0109.06001

[31] Lorentz, G. G.; Zeller, K. L., Birkhoff interpolation, SIAM J. Numer. Anal., 8, 43-48 (1971) · Zbl 0229.41001

[32] Ma, D.; Chen, J., Delay margin of low-order systems achievable by PID controllers, IEEE Trans. Autom. Control, 64, 5, 1958-1973 (2019) · Zbl 1482.93413

[33] Mallet-Paret, J.; Nussbaum, R. D., A differential-delay equation arising in optics and physiology, SIAM J. Math. Anal., 20, 2, 249-292 (1989) · Zbl 0676.34043

[34] Manitius, A. Z., Feedback controllers for a wind tunnel model involving a delay: analytical design and numerical simulation, IEEE Trans. Autom. Control, 29, 12, 1058-1068 (1984)

[35] Michiels, W.; Engelborghs, K.; Vansevenant, P.; Roose, D., Continuous pole placement for delay equations, Automatica, 38, 5, 747-761 (2002) · Zbl 1034.93026

[36] Michiels, W.; Niculescu, S.-I., Stability, Control, and Computation for Time-Delay Systems: An Eigenvalue-Based Approach, Advances in Design and Control, vol. 27 (2014), Society for Industrial and Applied Mathematics (SIAM): Society for Industrial and Applied Mathematics (SIAM) Philadelphia, PA

[37] Neĭmark, Y. I., The structure of the D-decomposition of the space of quasipolynomials and the diagrams of Vyšnegradskiĭ and Nyquist, Dokl. Akad. Nauk SSSR (N.S.), 60, 1503-1506 (1948)

[38] Niculescu, S.-I.; Michiels, W., Stabilizing a chain of integrators using multiple delays, IEEE Trans. Autom. Control, 49, 5, 802-807 (2004) · Zbl 1365.93412

[39] (Olver, F. W.J.; Lozier, D. W.; Boisvert, R. F.; Clark, C. W., NIST Handbook of Mathematical Functions. U.S. Department of Commerce (2010), National Institute of Standards and Technology: National Institute of Standards and Technology Washington, DC), Cambridge University Press, Cambridge, With 1 CD-ROM (Windows, Macintosh and UNIX) · Zbl 1198.00002

[40] Pinney, E., Ordinary Difference-Differential Equations (1958), University of California Press: University of California Press Berkeley-Los Angeles · Zbl 0091.07901

[41] Pólya, G.; Szegő, G., Problems and theorems in analysis. I, (Series, Integral Calculus, Theory of Functions. Series, Integral Calculus, Theory of Functions, Classics in Mathematics (1998), Springer-Verlag: Springer-Verlag Berlin), Translated from the German by Dorothee Aeppli, Reprint of the 1978 English translation · Zbl 0338.00001

[42] Ramírez, A.; Mondié, S.; Garrido, R.; Sipahi, R., Design of proportional-integral-retarded (PIR) controllers for second-order LTI systems, IEEE Trans. Autom. Control, 61, 6, 1688-1693 (2016) · Zbl 1359.93306

[43] Ronkin, A. L., Quasipolynomials, Funkc. Anal. Prilozh., 12, 4, 93-94 (1978) · Zbl 0423.32003

[44] Rouillier, F.; Din, M. S.E.; Schost, É., Solving the Birkhoff interpolation problem via the critical point method: an experimental study, (Automated Deduction in Geometry (2001), Springer Berlin Heidelberg), 26-40 · Zbl 0985.68079

[45] Rudin, W., Principles of Mathematical Analysis, International Series in Pure and Applied Mathematics (1976), McGraw-Hill Book Co.: McGraw-Hill Book Co. New York-Auckland-Düsseldorf · Zbl 0148.02903

[46] Sipahi, R.; Niculescu, S.-I.; Abdallah, C. T.; Michiels, W.; Gu, K., Stability and stabilization of systems with time delay: limitations and opportunities, IEEE Control Syst. Mag., 31, 1, 38-65 (2011) · Zbl 1395.93271

[47] Stavskiĭ, M.Š., A division algorithm with remainder for quasipolynomials, Barnaul. Gos. Ped. Inst. Učen. Zap., 9, 11-14 (1968)

[48] Stépán, G., Retarded Dynamical Systems: Stability and Characteristic Functions, Pitman Research Notes in Mathematics Series, vol. 210 (1989), Longman Scientific \& Technical: Longman Scientific \& Technical Harlow, copublished in the United States with John Wiley \& Sons, Inc., New York · Zbl 0686.34044

[49] Tripp, J. S., Development of a distributed parameter mathematical model for simulation of cryogenic wind tunnels (September 1983), NASA, Technical Paper 2177

[50] Vanbiervliet, J.; Verheyden, K.; Michiels, W.; Vandewalle, S., A nonsmooth optimisation approach for the stabilisation of time-delay systems, ESAIM Control Optim. Calc. Var., 14, 3, 478-493 (2008) · Zbl 1146.65056

[51] Čebotarev, N. G.; Meĭman, N. N., The Routh-Hurwitz problem for polynomials and entire functions. Real quasipolynomials with \(r = 3, s = 1\), Tr. Mat. Inst. Steklova, 26, 331 (1949), Appendix by G.S. Barhin and A.N. Hovanskiĭ

[52] Wielonsky, F., A Rolle's theorem for real exponential polynomials in the complex domain, J. Math. Pures Appl. (9), 80, 4, 389-408 (2001) · Zbl 1030.30024

[53] Wright, E. M., Stability criteria and the real roots of a transcendental equation, J. Soc. Ind. Appl. Math., 9, 136-148 (1961) · Zbl 0109.04705

[54] Wynn, P., On the zeros of certain confluent hypergeometric functions, Proc. Am. Math. Soc., 40, 173-182 (1973) · Zbl 0268.33004