Mastrostefano, Daniele


Summary: For every positive integer $N$ and every $\alpha \in [0,1)$, let $B(N, \alpha)$ denote the probabilistic model in which a random set $A \subset \{1, \ldots, N\}$ is constructed by choosing independently every element of $\{1, \ldots, N\}$ with probability $\alpha$. We prove that, as $N \to +\infty$, for every $A$ in $B(N, \alpha)$ we have $|AA| \sim |A|^2/2$ with probability $1 - o(1)$, if and only if

$$\frac{\log(\alpha^2 \log N \log 4 - 1)}{\sqrt{\log \log N}} \to -\infty.$$ 

This improves on a theorem of Cilleruelo, Ramana and Ramaré [J. Cilleruelo et al., Proc. Steklov Inst. Math. 296, 52–64 (2017; Zbl 1371.11023); translation in Tr. Mat. Inst. Steklova 296, 58–71 (2017)], who proved the above asymptotic between $|AA|$ and $|A|^2/2$ when $\alpha = o(1/\sqrt{\log N})$, and supplies a complete characterization of maximal product sets of random sets.

MSC:
11B30 Arithmetic combinatorics; higher degree uniformity

Keywords:
product sets; random models; localised divisor functions; distribution of the number of prime factors

Full Text: DOI arXiv

References:

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