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Bounding cubic-triple product Selmer groups of elliptic curves. (English) Zbl 1472.11182
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Let F be a totally real cubic number field, let E be a modular elliptic curve defined over F and let $h^1(E)$ be its F -motive. Via multiplicative induction to \mathbb{Q} one gets a *cubic-triple product motive* $M(E) := (\otimes \text{Ind}_{\mathbb{Q}}^F h^1(E))(2)$ of dimension 8, whose p -adic realization is basically (a twist of) the multiplicative induction from F to \mathbb{Q} of the p -adic Tate module of E . To such an object one can attach a triple product L -function $L(s, M(E))$ with good meromorphic properties and a functional equation with central critical value in $s = 0$.

The paper deals with an instance of the Bloch-Kato conjecture for this setting: in particular, under some additional hypotheses on E , it proves that the nonvanishing of $L(0, M(E))$ yields the 0-dimensionality (over \mathbb{Q}_p) of the p -part of the appropriate Selmer group for infinitely many primes p . The author already presented a similar result, where F was replaced by the product of \mathbb{Q} and a real quadratic field, in [*Y. Lin*, *Invent. Math.* 205, No. 3, 693–780 (2016; [Zbl 1395.11091](#))]: the strategy is based on a reciprocity law for cycles on a triple product of modular curves, which is obtained via congruence formulas arising from computations of étale local cohomology groups of varieties. Though the strategy is similar to the Lin paper mentioned above, the different setting requires new techniques on weight spectral sequences to handle higher dimensional cycles, which then provide the cohomological computations (hence the reciprocity law) needed here.

When $L(0, M(E)) \neq 0$, the reciprocity law on some special (Hirzebruch-Zagier) cycles produces enough annihilators for the Selmer group to prove its finiteness.

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MSC:

[11G05](#) Elliptic curves over global fields
[11R34](#) Galois cohomology
[14G35](#) Modular and Shimura varieties

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