On the cut number problem for the 4, and 5-cubes. (English) Zbl 1472.52036

Summary: The hypercube cut number $S(d)$ is the minimum number of hyperplanes in the $d$-dimensional Euclidean space that slice all the edges of the $d$-cube. The determination of $S(d)$ in dimensions 5, 6, and 7, is one of the Victor Klee’s unresolved problems presented in one of his invited talks on problems from discrete geometry. The value of $S(d)$ is unknown for $d \geq 7$, but for $d \leq 3$ it is trivial to show $S(d) = d$. In the late nineteen eighties, Emamy-K has shown $S(4) = 4$ via two different proofs. More than a decade later, Sohler and Ziegler obtained a computational proof of $S(5) = 5$ that took about two months of CPU computing time. From $S(5) = 5$ it can be verified that $S(6) = 5$. A short mathematical proof for $d = 5$ remains to be a challenging problem that also leads one to the insight of the still open 7-dimensional problem. In this article we present a vertex coloring approach on the hypercube that gives a simplified proof for $d = 4$ and also helps toward a short mathematical proof for $d = 5$, free of computer computations.

MSC:
52C35 Arrangements of points, flats, hyperplanes (aspects of discrete geometry)

Keywords: convex and discrete geometry; hyperplanes; hypercube cuts; vertex coloring

Full Text: DOI

References:

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.