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**Sharp phase transition for the continuum Widom-Rowlinson model.**  (English. French summary)


Summary: The Widom-Rowlinson model (or the Area-interaction model) is a Gibbs point process in $\mathbb{R}^d$ with the formal Hamiltonian defined as the volume of $\bigcup_{x \in \omega} B_1(x)$, where $\omega$ is a locally finite configuration of points and $B_1(x)$ denotes the unit closed ball centred at $x$. The model is also tuned by two other parameters: the activity $z > 0$ related to the intensity of the process and the inverse temperature $\beta \geq 0$ related to the strength of the interaction. In the present paper we investigate the phase transition of the model in the point of view of percolation theory and the liquid-gas transition. First, considering the graph connecting points with distance smaller than $2r > 0$, we show that for any $\beta \geq 0$, there exists $0 < \tilde{z}_a(\beta, r) < +\infty$ such that an exponential decay of connectivity at distance $n$ occurs in the subcritical phase (i.e. $z < \tilde{z}_a(\beta, r)$) and a linear lower bound of the connection at infinity holds in the supercritical case (i.e. $z > \tilde{z}_a(\beta, r)$). These results are in the spirit of recent works using the theory of randomised tree algorithms [H. Duminil-Copin et al., Probab. Theory Relat. Fields 173, No. 1–2, 479–490 (2019; Zbl 1478.60259); Ann. Math. (2) 189, No. 1, 75–99 (2019; Zbl 1482.82009)]. Secondly we study a standard liquid-gas phase transition related to the uniqueness/non-uniqueness of Gibbs states depending on the parameters $z, \beta$. Old results [D. Ruelle, “Existence of a phase transition in a continuous classical system”, Phys. Rev. Lett. 27, 1040–1041 (1971; doi:10.1103/PhysRevLett.27.1040; A. Mazel et al., J. Stat. Phys. 159, No. 5, 1040–1086 (2015; Zbl 1329.82013)] claim that a non-uniqueness regime occurs for $z = \beta$ large enough and it is conjectured that the uniqueness should hold outside such an half line ($z = \beta \geq \beta_c > 0$). We solve partially this conjecture in any dimension by showing that for $\beta$ large enough the non-uniqueness holds if and only if $z = \beta$. We show also that this critical value $z = \beta$ corresponds to the percolation threshold $\tilde{z}_a(\beta, r) = \beta$ for $\beta$ large enough, providing a straight connection between these two notions of phase transition.

MSC:

- 60D05 Geometric probability and stochastic geometry
- 60G10 Stationary stochastic processes
- 60G55 Point processes (e.g., Poisson, Cox, Hawkes processes)
- 60G57 Random measures
- 60G60 Random fields
- 60K35 Interacting random processes; statistical mechanics type models; percolation theory
- 82B21 Continuum models (systems of particles, etc.) arising in equilibrium statistical mechanics
- 82B26 Phase transitions (general) in equilibrium statistical mechanics
- 82B43 Percolation

Keywords:

- Boolean model; continuum percolation; DLR equations; Fortuin-Kasteleyn representation; Gibbs point process; OSSS inequality; random cluster model; randomised tree algorithm

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References:


