Summary: For a dispersive partial differential equation, the degeneracy of its dispersion relation will deteriorate dispersion of waves and strengthen nonlinear effects. Such negative effects can sometimes be mitigated by some null structure in the nonlinearity. Motivated by water-wave problems, in this paper we consider a class of nonlinear dispersive PDEs in 2D with cubic nonlinearities, whose dispersion relations are radial and have vanishing Gaussian curvature on a circle. For such a model we identify certain null structures for the cubic nonlinearity, which suffice in order to guarantee global scattering solutions for the small data problem. Our null structures in the power-type nonlinearity are weak and only eliminate the worst nonlinear interaction. Such null structures arise naturally in some water-wave problems.

MSC:

35A01 Existence problems for PDEs: global existence, local existence, non-existence
35A02 Uniqueness problems for PDEs: global uniqueness, local uniqueness, non-uniqueness
35G25 Initial value problems for nonlinear higher-order PDEs
35Q55 NLS equations (nonlinear Schrödinger equations)

Keywords:

localized Strichartz estimates; nonlinear dispersive PDEs in 2D; cubic nonlinearities; global well-posedness for small data

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