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For coprime integers $p$ and $q$, the lens space $L(p,q)$ is the quotient of $S^3$ by the $\mathbb{Z}/p\mathbb{Z}$-action $(z_0, z_1) \mapsto (e^{2\pi i/p} z_0, e^{2\pi q/p} z_1)$. It admits a CW-structure $P^2(p) \cup e^3$ where $P^2(p)$ is the mapping cone of the degree map $p: S^1 \to S^1$.

Let $P$ be a principal $U(n)$-bundle over $L(p,q)$ where $p$ is a prime. The isomorphism class of $P$ is determined by its first Chern class $k \in H^2(L(p,q); \mathbb{Z})$ which is $\mathbb{Z}/p\mathbb{Z}$. The gauge group of $P$, denoted by $G_k(L(p,q))$, is the topological group consisting of $U(n)$-equivariant automorphisms of $P$ that fix $L(p,q)$. It is known that gauge groups of $U(n)$-bundles over $L(p,q)$ have finitely many distinct homotopy types. Classifying the homotopy types of various $G_k(L(p,q))$ is important to understand the topology of these gauge groups.

In Section 2 the authors study the homotopy theory of $L(p,q)$ and show that the inclusion $P^2(p) \to L(p,q)$ induces an isomorphism between $[L(p,q), BU(n)]$ and $[P^2(p), BU(n)]$ which are $\mathbb{Z}/p\mathbb{Z}$.

Denote by $G_k(P^2(p))$ the gauge group of the principal $U(n)$-bundle over $P^2(p)$ with first Chern class $k \in H^2(P^2(p); \mathbb{Z}) \cong \mathbb{Z}/p\mathbb{Z}$. In Section 3 the authors classify the homotopy types of various $G_k(P^2(p))$ and show that $G_k(P^2(p)) \simeq G(2)P(p)$ if and only if $gcd(p,k) = gcd(p,\ell)$.

Combining their work in Sections 2 and 3, in Section 4 the authors prove that $gcd(p,k) = gcd(p,\ell)$ if $G_k(L(p,q)) \simeq G_k(L(p,q))$. Moreover, they show the converse whenever there exists an integer $u$ such that $k \equiv ul \pmod{p}$ and $u^2 \equiv \pm 1 \pmod{p}$. In particular, for $p \in \{3, 5\}$, $G_k(L(p,q)) \simeq G(2)(L(p,q))$ if and only if $gcd(p,k) = gcd(p,\ell)$.

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References:


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