In this paper a conjecture of Fox, Huang, and Lee is proved that characterizes directed graphs having constant density in all tournaments. A digraph $\overrightarrow{H}$ is said to be impartial if it has the following property. For some $n$ at least the order of $\overrightarrow{H}$, all $n$-vertex tournaments contain the same number of copies of $\overrightarrow{H}$ as subgraphs. A digraph $\overrightarrow{T}$ is said to be recursively bridge-mirrored if it can be constructed recursively in the following manner from a single vertex. Mark an arbitrary vertex of $\overrightarrow{T}$ as its root, and create a new graph by taking two identical copies of this rooted $\overrightarrow{T}$ and adding a new directed edge from one root to the other. The main theorem established is the following equivalence. A directed graph is impartial if and only if it is a disjoint union of recursively bridge-mirrored digraphs.

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References: