Blekherman, Grigoriy; Raymond, Annie; Singh, Mohit; Thomas, Rekha R.

Simple graph density inequalities with no sum of squares proofs. (English) [Zbl 1474.05210]

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Let \( H \) and \( G \) be graphs without loops and multiple edges. Then a mapping \( f : V(H) \to V(G) \) is called a graph homomorphism if the map induced by \( f \) assigns to each edge of \( H \) an edge of \( G \). The probability of a random mapping from \( V(H) \) to \( V(G) \) to be a graph homomorphism, denoted by \( t(H; G) \), is called the homomorphism density of a graph \( H \) in a graph \( G \). An expression containing homomorphism densities is non-negative if it holds for all graphs \( G \). A standard method to prove the non-negativity of a density expression is to write it as a sum of squares. The main result of this paper provides a simple sufficient condition for a density expression not to be representable as a sum of squares. Using this result, the authors show that the non-negativity of some non-negative density expressions cannot be proved by the sum of squares method. These results answer in the affirmative two questions raised by Lovász.

Reviewer: Peter Horák (Tacoma)

MSC:

05C35 Extremal problems in graph theory
90C22 Semidefinite programming
90C35 Programming involving graphs or networks

Keywords:

homomorphism density; sum of squares

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References:

[12] Li, J. L. X.; Szegedy, B., On the logarithmic calculus and Sidorenko’s conjecture (2011)


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