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Cycle lengths in expanding graphs. (English) [Zbl 1474.05220]

For a positive constant $\alpha$ a graph $G$ on $n$ vertices is called an $\alpha$-expander if every vertex set $U$ of size at most $n/2$ has an external neighborhood whose size is at least $\alpha|U|$. It is proved that cycle lengths in $\alpha$-expanders are well distributed. In particular, it is shown that for every $0 < \alpha \leq 1$ there exist positive constants $n_0$, $C$ and $A = O(1/\alpha)$ such that for every $\alpha$-expander $G$ on $n \geq n_0$ vertices and every integer $\ell \in [C\log n, \frac{n}{\alpha}]$, $G$ contains a cycle whose length is between $\ell$ and $\ell + A$; the order of dependence of the additive error term $A$ on $\alpha$ is optimal. Secondly, it is shown that every $\alpha$-expander on $n$ vertices contains $\Omega\left(\frac{n^3}{\log(1/\alpha)}\right)$ different cycle lengths. Finally, it is introduced another expansion-type property, guaranteeing the existence of a linearly long interval in the set of cycle lengths. Namely, for $\beta > 0$ a graph $G$ on $n$ vertices is called $\beta$-graph if every pair of disjoint sets of size at least $\beta n$ are connected by an edge. It is proved that for every $\beta < 1/20$ there exist positive constants $b_1 = O\left(\frac{1}{\log(1/\beta)}\right)$ and $b_2 = O(\beta)$ such that every $\beta$-graph on $n$ vertices contains a cycle of length $\ell$ for every integer $\ell \in [b_1 \log n, (1 - b_2)n]$; the order of dependence of $b_1$ and $b_2$ on $\beta$ is optimal.

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05C38 Paths and cycles
05C48 Expander graphs
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