For positive integers \( a > b \), an \((a, b)\)-graph \( G = (V, E) \) is an \( a \)-regular graph in which for every \( v \in V \) the link \( G_v \), i.e. the subgraph induced by the set of neighbours of \( v \), is \( b \)-regular. We are interested in the expansion properties of \((a, b)\)-graphs, noting that for a number of the many applications of expander graphs (e.g. in PCP theory on computer science) we want not only the basic graph \( G \) to have good expansion properties but also the link graphs \( G_v \). (A random regular graph is, with probability tending to 1 a good expander in a suitable sense: but \( G_v \) is typically an anticlique so not an expander.)

The paper under review constructs two families of \((a, b)\)-graphs where both \( G \) and the links \( G_v \) have good expansion properties. The authors consider the second eigenvalue \( \lambda_2 \) of the graph \( G \), where it is well known that comparatively small values of \( \lambda_2 \) correspond to good expansion properties. An initial result, based on a result of N. Alon and R. B. Boppana [Combinatorica 7, 1–22 (1987; Zbl 0631.68041)], is that the second eigenvalue of an \((a, b)\)-graph is at least \( b + 2\sqrt{a-b-1} + o(1) \). This bound is tight.

However there is a degree of ‘trade-off’ between good expansion of \( G_v \) and \( G \). in that as the quality of the expansion in \( G_v \) increases the gap between the second eigenvalue of \( G \) and the lower bound just mentioned increases. A key role in constructing the graphs is played by the so-called polygraph construction, which (crudely speaking) transforms high-girth expanders into \((a, b)\)-graphs.

This work fits into the active research topic of high-dimensional expansion, and there is discussion of links with this area in Section 5 of the paper. Work is still proceeding in this area: see, for example, the recent preprint of E. Friedgut and Y. Iluz [“Hyper-regular graphs and high dimensional expanders”, Preprint, arxiv:2010.03829] in which related results are given and relevant literature surveyed. Some other classes of \((a, b)\)-regular graphs, based on regular triangulations of surfaces and tensor products of graphs. A number of open problems are also listed. Including quantifying more precisely the trade-off mentioned above.

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