Aliev, Rashid Avazaga; Ahmadova, Aynur Nofel

Boundedness of discrete Hilbert transform on discrete Morrey spaces. (English)

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Summary: The Hilbert transform plays an important role in the theory and practice of signal processing operations in continuous system theory because of its relevance to such problems as envelope detection and demodulation, as well as because of its use in relating the real and imaginary components, and the magnitude and phase components of spectra. The Hilbert transform is a multiplier operator and is widely used in the theory of Fourier transforms. The Hilbert transform was the motivation for the development of modern harmonic analysis. Its discrete version is also widely used in many areas of science and technology and plays an important role in digital signal processing. The essential motivation behind thinking about discrete transforms is that experimental data are most frequently not taken in a continuous manner but sampled at discrete time values. Since much of the data collected in both the physical sciences and engineering are discrete, the discrete Hilbert transform is a rather useful tool in these areas for the general analysis of this type of data.

The Hilbert transform has been well studied on classical function spaces Lebesgue, Morrey, etc. But its discrete version, which also has numerous applications, has not been fully studied in discrete analogues of these spaces. In this paper we discuss the discrete Hilbert transform on discrete Morrey spaces. In particular, we obtain its boundedness on the discrete Morrey spaces using boundedness of the Hilbert transform on Morrey spaces.

MSC:

44A15 Special integral transforms (Legendre, Hilbert, etc.)
39A12 Discrete version of topics in analysis
46B45 Banach sequence spaces
42B35 Function spaces arising in harmonic analysis

Keywords:

discrete Hilbert transform; Morrey spaces; discrete Morrey spaces; boundedness

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