Scheucher, Manfred
Two disjoint 5-holes in point sets. (English) Zbl 1474.68428

Summary: Given a set of points $S \subseteq \mathbb{R}^2$, a subset $X \subseteq S$ with $|X| = k$ is called $k$-gon if all points of $X$ lie on the boundary of the convex hull of $X$, and $k$-hole if, in addition, no point of $S \setminus X$ lies in the convex hull of $X$. We use computer assistance to show that every set of 17 points in general position admits two disjoint 5-holes, that is, holes with disjoint respective convex hulls. This answers a question of K. Hosono and M. Urabe [Comput. Geom. 20, No. 3, 97–104 (2001; Zbl 0990.68171)]. We also provide new bounds for three and more pairwise disjoint holes.

In a recent article, K. Hosono and M. Urabe [AKCE Int. J. Graphs Comb. 17, No. 1, 7–15 (2020; Zbl 1475.52027)] present new results on interior-disjoint holes – a variant, which also has been investigated in the last two decades. Using our program, we show that every set of 15 points contains two interior-disjoint 5-holes.

Moreover, our program can be used to verify that every set of 17 points contains a 6-gon within significantly smaller computation time than the original program by G. Szekeres and L. Peters [ANZIAM J. 48, No. 2, 151–164 (2006; Zbl 1152.52008)]. Another independent verification of this result was done by F. Marić [J. Autom. Reasoning 62, No. 3, 301–329 (2019; Zbl 1468.68302)].

MSC:
68U05 Computer graphics; computational geometry (digital and algorithmic aspects)
52B55 Computational aspects related to convexity
52C10 Erdős problems and related topics of discrete geometry
68V05 Computer assisted proofs of proofs-by-exhaustion type

Keywords:
Boolean satisfiability (SAT); empty convex polygon; Erdős-Szekeres-type problem

Software:
PicoSAT; DRAT-trim

Full Text: DOI arXiv

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