The article under review contributes to research based around Rademacher’s theorem, which states that a Lipschitz mapping between two Euclidean spaces is differentiable almost everywhere with respect to the Lebesgue measure. More specifically, the article joins a flourishing branch of research which has the broad aim of extending Rademacher’s theorem to more general settings; see for example [J. Cheeger and B. Kleiner, Geom. Funct. Anal. 19, No. 4, 1017–1028 (2009; Zbl 1200.58007)] and [D. Bate and S. Li, Adv. Math. 333, 868–930 (2018; Zbl 1402.30047)].

Extensions of Rademacher’s theorem to infinite dimensional domains and metric measure space domains without a vector space structure must overcome various challenges. Since there is no Lebesgue like measure, more specifically, the article joins a flourishing branch of research which has the broad aim of extending Rademacher’s theorem to more general settings; see for example [J. Cheeger and B. Kleiner, Geom. Funct. Anal. 19, No. 4, 1017–1028 (2009; Zbl 1200.58007)] and [D. Bate and S. Li, Adv. Math. 333, 868–930 (2018; Zbl 1402.30047)]. The article under review defines Gâteaux differentiability of a function defined on a suitable metric scalable group and every point outside of

\[(i)\] \[\delta_\lambda := \delta(\lambda, \cdot) \in \text{Aut}(G)\text{ for every } \lambda \in \mathbb{R} \setminus \{0\}.\]

\[(ii)\] \[\delta_\lambda \circ \delta_\mu = \delta_{\lambda \mu}\text{ for all } \lambda, \mu \in \mathbb{R}.\]

\[(iii)\] \[\delta_0 \equiv e_G,\text{ where } e_G\text{ is the identity element of } G.\]

A scalable group \((G, \delta)\) is called a metric scalable group if it admits an admissable left invariant metric \(d\), such that

\[d(\delta_t(p), \delta_t(q)) = |t|d(p, q)\]

for all \(t \in \mathbb{R}\) and all \(p, q \in G\). Finally a complete metric scalable group \(G\) is called an infinite dimensional Carnot group if it admits a sequence \((N_m)_{m \in \mathbb{N}}\) of scalable subgroups \(N_m\) such that each \(N_m\) has a Carnot group structure, \(N_m < N_{m+1}\) and \(G\) is the closure of \(\bigcup_{m \in \mathbb{N}} N_m\). In this case we say that \((N_m)_{m \in \mathbb{N}}\) is a filtration by Carnot subgroups of \(G\). The article under review establishes an extension of Rademacher’s theorem to infinite dimensional Carnot group domains.

The appropriate \(\sigma\)-ideal of null sets in an infinite dimensional Carnot group \(G\) is obtained via the notion of filtration negligible sets: Given a filtration \((N_m)_{m \in \mathbb{N}}\) by Carnot subgroups of \(G\), a Borel set \(\Omega \subseteq G\) is called \((N_m)_{m \in \mathbb{N}}\)-negligible if it is the countable union of Borel sets \(\Omega_m\) with

\[\text{vol}_{N_m}(N_m \cap (g\Omega_m)) = 0\]

for every \(m \in \mathbb{N}\) and every \(g \in G\). Here, \(\text{vol}_{N_m}\) denotes an arbitrarily chosen Haar measure on \(N_m\).

To conclude, the authors prove that any real-valued Lipschitz function \(f\) defined on an infinite dimensional Carnot group admits a Borel set \(\Omega \subseteq G\), such that \(f\) is Gâteaux-differentiable, in the sense of Pansu, at every point outside of \(\Omega\) and \(\Omega\) is \((N_m)_{m \in \mathbb{N}}\)-negligible with respect to every filtration \((N_m)_{m \in \mathbb{N}}\) by Carnot subgroups of \(G\).

Reviewer: Michael Dymond (Leipzig)
MSC:
28A15 Abstract differentiation theory, differentiation of set functions
46G05 Derivatives of functions in infinite-dimensional spaces
53C17 Sub-Riemannian geometry
58C20 Differentiation theory (Gateaux, Fréchet, etc.) on manifolds

Cited in 3 Documents

Keywords:
Carnot groups; differentiability; Rademacher theorem; Gâteaux derivative

Full Text: DOI

References:


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