A diffeomorphism-invariant metric on the space of vector-valued one-forms. (English)

The article under review introduces a diffeomorphism-invariant Riemannian metric on the space of vector-valued one-forms on a compact (orientable) manifold. As this metric does not involve derivation of vector component, the authors call this an $L^2$-type metric. Moreover, the new metric is connected to the Ebin metric (cf. [D. G. Ebin, in: Global Analysis, Proc. Sympos. Pure Math. 15, 11–40 (1970; Zbl 0205.53702)]) on the space of all Riemannian metrics via a Riemannian submersion.

In a bit more detail, the authors consider the manifold of vector-valued one-forms of full rank (where a one form is of full rank if it is injective). Then the geodesic equations are calculated. Recall that on an infinite-dimensional manifold the geodesic distance may be degenerate and thus fail to be a metric (see [P. W. Michor and D. Mumford, Doc. Math. 10, 217–245 (2005; Zbl 1083.58010)]). In the case at hand, the geodesic equations admit explicit solution formulae and an explicit computation of the geodesics. As a consequence, it is proved that the geodesic distance is non-degenerate. Furthermore, the authors prove that the Riemannian metric leads to a geodesically and metrically incomplete space. Then completely geodesic subspaces are studied. Finally, the sectional curvature of the infinite-dimensional Riemannian metric is calculated. Here the main observation is that, depending on the dimension of the base manifold and the target space, the sectional curvature either has a semidefinite sign or admits both signs.

Reviewer: Alexander Schmeding (Bods)

MSC:
58D15 Manifolds of mappings
58B20 Riemannian, Finsler and other geometric structures on infinite-dimensional manifolds
58D17 Manifolds of metrics (especially Riemannian)

Keywords:
space of Riemannian metrics; Ebin metric; sectional curvature; shape analysis

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