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**Chaotic oscillations of linear hyperbolic PDE with variable coefficients and implicit boundary conditions.** (English) [Zbl 1476.34106](#)

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**Summary:** In this paper, the chaotic oscillations of the initial-boundary value problem of linear hyperbolic partial differential equation (PDE) with variable coefficients are investigated, where both ends of boundary conditions are nonlinear implicit boundary conditions (IBCs). It separately considers that IBCs can be expressed by general nonlinear boundary conditions (NBCs) and cannot be expressed by explicit boundary conditions (EBCs). Finally, numerical examples verify the effectiveness of theoretical prediction.

**MSC:**

**34C28** Complex behavior and chaotic systems of ordinary differential equations

**35L70** Second-order nonlinear hyperbolic equations








**35L05** Wave equation

**Keywords:**

chaotic oscillation; hyperbolic PDE; variable coefficient; implicit boundary condition; nonlinear boundary condition

**Full Text:** [DOI](#)

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