Summary: Let \( \pi : \mathbb{R}^n \to \mathbb{R}^d \) be any linear projection, let \( A \) be the image of the standard basis. Motivated by Postnikov’s study of positive Grassmannians via plabic graphs and Galashin’s connection of plabic graphs to slices of zonotopal tilings of 3-dimensional cyclic zonotopes, we study the poset of subdivisions induced by the restriction of \( \pi \) to the \( k \)-th hypersimplex, for \( k = 1, \ldots, n-1 \). We show that: For arbitrary \( A \) and for \( k \leq d + 1 \), the corresponding fiber polytope \( \mathcal{F}(k)(A) \) is normally isomorphic to the Minkowski sum of the secondary polytopes of all subsets of \( A \) of size \( \max\{d+2, n-k+1\} \). When \( A = P_n \) is the vertex set of an \( n \)-gon, we answer the Baues question in the positive: the inclusion of the poset of \( \pi \)-coherent subdivisions into the poset of all \( \pi \)-induced subdivisions is a homotopy equivalence. When \( A = C(d, n) \) is the vertex set of a cyclic \( d \)-polytope with \( d \) odd and any \( n \geq d + 3 \), there are non-lifting (and even more so, non-separated) \( \pi \)-induced subdivisions for \( k = 2 \).

MSC:

- 52B20 Lattice polytopes in convex geometry (including relations with commutative algebra and algebraic geometry)
- 52B45 Dissections and valuations (Hilbert’s third problem, etc.)
- 52C22 Tilings in \( n \) dimensions (aspects of discrete geometry)
- 52C40 Oriented matroids in discrete geometry
- 51M20 Polyhedra and polytopes; regular figures, division of spaces
- 05C10 Planar graphs; geometric and topological aspects of graph theory
- 05C15 Coloring of graphs and hypergraphs

Keywords:

hypersimplex; subdivisions; fiber polytope; Baues problem; separated sets; Zitat Galashin: 1406.52039; plabic graphs; plabic=planar bicolored

Full Text: DOI

References:
