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On the optimal map in the 2-dimensional random matching problem. (English)

Summary: We show that, on a 2-dimensional compact manifold, the optimal transport map in the semi-discrete random matching problem is well-approximated in the $L^2$-norm by identity plus the gradient of the solution to the Poisson problem $-\Delta f^{n,t} = \mu^{n,t} - 1$, where $\mu^{n,t}$ is an appropriate regularization of the empirical measure associated to the random points. This shows that the ansatz of [S. Caracciolo et al., “Scaling hypothesis for the Euclidean bipartite matching problem”, Physical Review E 90, 012118 (2014)] is strong enough to capture the behavior of the optimal map in addition to the value of the optimal matching cost.

As part of our strategy, we prove a new stability result for the optimal transport map on a compact manifold.

MSC:
60D05 Geometric probability and stochastic geometry
35B35 Stability in context of PDEs
35F21 Hamilton-Jacobi equations
49J55 Existence of optimal solutions to problems involving randomness
49Q20 Variational problems in a geometric measure-theoretic setting
58J35 Heat and other parabolic equation methods for PDEs on manifolds

Keywords: minimum matching; random matching; optimal transport; Hamilton-Jacobi; stability

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