Reducing graph transversals via edge contractions. (English) [Zbl 1477.68239]

Summary: For a graph invariant \( \pi \), the CONTRACTION(\( \pi \)) problem consists of, given a graph \( G \) and positive integers \( k, d \), deciding whether one can contract \( k \) edges of \( G \) to obtain a graph in which \( \pi \) has dropped by at least \( d \). E. Galby et al. [Discrete Math. 344, No. 1, Article ID 112169, 26 p. (2021; Zbl 1455.05055); Theor. Comput. Sci. 877, 18–35 (2021; Zbl 1478.68239)] studied the case where \( \pi \) is the size of a minimum dominating set. We focus on graph invariants defined as the minimum size of a vertex set that hits all the occurrences of graphs in a collection \( H \) according to a fixed containment relation. We prove co-NP-hardness results under some assumptions on the graphs in \( H \), in particular implying that CONTRACTION(\( \pi \)) is co-NP-hard for fixed \( k = d = 1 \) when \( \pi \) is the size of a minimum feedback vertex set or an odd cycle transversal. In sharp contrast, when \( \pi \) is the size of a minimum vertex cover, the problem is in XP parameterized by \( d \).

MSC:
68R10 Graph theory (including graph drawing) in computer science
68Q17 Computational difficulty of problems (lower bounds, completeness, difficulty of approximation, etc.)
68Q27 Parameterized complexity, tractability and kernelization

Keywords:
blocker problem; edge contraction; graph transversal; parameterized complexity; vertex cover; feedback vertex set; odd cycle transversal

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