

Orr, Martin

Unlikely intersections with Hecke translates of a special subvariety. (English) Zbl 1478.11089
J. Eur. Math. Soc. (JEMS) 23, No. 1, 1-28 (2021).

This paper discusses some cases of the Zilber-Pink conjecture. Specifically, Conjecture 1.2 in the paper looks at the restriction of Zilber-Pink to look at the intersections of an irreducible algebraic curve $V \subset S$ which is not contained in any proper special subvariety of S , with a special subvariety $S_{\mathbf{H}} \subset S$ of codimension at least 2 (here S is a pure Shimura variety).

On one hand the paper obtains a conditional result, where Conjecture 1.2 is proven under two arithmetic conjectures. This is not an uncommon feature in the area of Zilber-Pink problems as the proofs usually rely on the well-established Pila-Zannier strategy which combines o-minimality with point counting, and it is now well-understood that the arithmetic part is usually more difficult.

But the paper also proves two unconditional cases of Conjecture 1.2 when the Shimura variety is of the form $\mathcal{A}_g \times \mathcal{A}_g$, where $g \geq 2$ and \mathcal{A}_g is the moduli space of principally polarised abelian varieties of dimension g . Theorems 1.3 and 1.4 are the first proven cases of Zilber-Pink that do not fall under the setting of products of modular curves, or of the Andr e-Oort conjecture.

Aside from using the Pila-Zannier strategy, the proofs use previous results of the author and some recent functional transcendence results of Z. Gao [*J. Reine Angew. Math.* 732, 85–146 (2017; [Zbl 1422.11140](#))].

Reviewer: [Sebastian Eterovi c \(Berkeley\)](#)

MSC:

[11G18](#) Arithmetic aspects of modular and Shimura varieties
[14G35](#) Modular and Shimura varieties

Cited in 1 Document

Keywords:

[unlikely intersections](#); [Shimura varieties](#); [Hecke correspondences](#); [Zilber-Pink](#)

Full Text: [DOI](#) [arXiv](#)

References:

- [1] Abramovich, D.: A linear lower bound on the gonality of modular curves. *Int. Math. Res. Notices*1996, 1005-1011Zbl 0878.14019 MR 1422373 · [Zbl 0878.14019](#)
- [2] Andr e, Y.: Mumford-Tate groups of mixed Hodge structures and the theorem of the fixed part. *Compos. Math.*82, 1-24 (1992)Zbl 0770.14003 MR 1154159
- [3] Andrianov, A. N.: Quadratic Forms and Hecke Operators. Grundlehren Math. Wiss. 286, Springer, Berlin (1987)Zbl 0613.10023 MR 0884891 · [Zbl 0613.10023](#)
- [4] Baily, W. L., Jr., Borel, A.: Compactification of arithmetic quotients of bounded symmetric domains. *Ann. of Math. (2)*84, 442-528 (1966)Zbl 0154.08602 MR 0216035 · [Zbl 0154.08602](#)
- [5] Borel, A.: Introduction aux groupes arithm etiques. *Publ. Inst. Math. Univ. Strasbourg XV, Act. Sci. Indust.* 1341, Hermann, Paris (1969)Zbl 0186.33202 MR 0244260 · [Zbl 0186.33202](#)
- [6] Borovoi, M. V.: Langlands' conjecture concerning conjugation of connected Shimura varieties. *Selecta Math. Soviet.*3, 3-39 (1983/84)Zbl 0555.32020 MR 0732450 · [Zbl 0555.32020](#)
- [7] Daw, C., Ren, J.: Applications of the hyperbolic Ax-Schanuel conjecture. *Compos. Math.*154, 1843-1888 (2018)Zbl 1427.11057 MR 3867286 · [Zbl 1427.11057](#)
- [8] Deligne, P.: Travaux de Shimura. In: *S eminaire Bourbaki (1970/71)*, exp. 389, Lecture Notes in Math. 244, Springer, 123-165 (1971)Zbl 0225.14007 MR 0498581
- [9] Deligne, P.: Vari et es de Shimura: interpr etation modulaire, et techniques de construction de mod eles canoniques. In: *Automorphic Forms, Representations and L-functions (Part 2)*, Proc. Sympos. Pure Math. 33, Amer. Math. Soc., Providence, RI, 247-289 (1979)Zbl 0437.14012 MR 0546620
- [10] Edixhoven, B., Yafaev, A.: Subvarieties of Shimura varieties. *Ann. of Math. (2)*157, 621-645 (2003)Zbl 1053.14023 MR 1973057 · [Zbl 1053.14023](#)

- [11] Ellenberg, J. S., Hall, C., Kowalski, E.: Expander graphs, gonality, and variation of Galois representations. *Duke Math. J.* 161, 1233-1275 (2012) Zbl 1262.14021 MR 2922374 · Zbl 1262.14021
- [12] Faltings, G.: Endlichkeitssätze für abelsche Varietäten über Zahlkörpern. *Invent. Math.* 73, 349-366 (1983) Zbl 0588.14026 MR 0718935
- [13] Gao, Z.: Towards the André-Oort conjecture for mixed Shimura varieties: the Ax-Lindemann theorem and lower bounds for Galois orbits of special points. *J. Reine Angew. Math.* 732, 85-146 (2017) Zbl 1422.11140 MR 3717089 · Zbl 1422.11140
- [14] Habegger, P., Pila, J.: Some unlikely intersections beyond André-Oort. *Compos. Math.* 148, 1-27 (2012) Zbl 1288.11062 MR 2881307 · Zbl 1288.11062
- [15] Habegger, P., Pila, J.: O-minimality and certain atypical intersections. *Ann. Sci. École Norm. Sup. (4)* 49, 813-858 (2016) Zbl 1364.11110 MR 3552014 · Zbl 1364.11110
- [16] Hindry, M., Silverman, J. H.: *Diophantine Geometry: An Introduction*. Grad. Texts in Math. 201, Springer, New York (2000) Zbl 0948.11023 MR 2223493 · Zbl 0948.11023
- [17] Klingler, B., Ullmo, E., Yafaev, A.: The hyperbolic Ax-Lindemann-Weierstrass conjecture. *Publ. Math. Inst. Hautes Études Sci.* 123, 333-360 (2016) Zbl 1372.14016 MR 3502100 · Zbl 1372.14016
- [18] Masser, D., Wüstholz, G.: Isogeny estimates for abelian varieties, and finiteness theorems. *Ann. of Math. (2)* 137, 459-472 (1993) Zbl 0804.14019 MR 1217345 · Zbl 0804.14019
- [19] Milne, J. S.: The action of an automorphism of \mathbb{C} on a Shimura variety and its special points. In: *Arithmetic and Geometry, Vol. I*, Progr. Math. 35, Birkhäuser Boston, Boston, MA, 239-265 (1983) Zbl 0527.14035 MR 0717596
- [20] Mok, N., Pila, J., Tsimerman, J.: Ax-Schanuel for Shimura varieties. *Ann. of Math. (2)* 189, 945-978 (2019) Zbl 07107179 MR 3961087 · Zbl 1481.14048
- [21] Moonen, B.: Linearity properties of Shimura varieties. I. *J. Algebraic Geom.* 7, 539-567 (1998) Zbl 0956.14016 MR 1618140 · Zbl 0956.14016
- [22] Newman, M., Smart, J. R.: Symplectic modular groups. *Acta Arith.* 9, 83-89 (1964) Zbl 0135.06502 MR 0162862 · Zbl 0135.06502
- [23] Nori, M. V.: On subgroups of $GL_n(\mathbb{F}_p)$. *Invent. Math.* 88, 257-275 (1987) Zbl 0632.20030 MR 0880952 · Zbl 0632.20030
- [24] Orr, M.: Families of abelian varieties with many isogenous fibres. *J. Reine Angew. Math.* 705, 211-231 (2015) Zbl 1349.14143 MR 3377393 · Zbl 1349.14143
- [25] Orr, M.: Height bounds and the Siegel property. *Algebra Number Theory* 12, 455-478 (2018) Zbl 06880895 MR 3803710 · Zbl 1442.11074
- [26] Pila, J.: On a modular Fermat equation. *Comment. Math. Helv.* 92, 85-103 (2017) Zbl 1410.11074 MR 3615036 · Zbl 1410.11074
- [27] Pila, J., Zannier, U.: Rational points in periodic analytic sets and the Manin-Mumford conjecture. *Rend. Lincei Mat. Appl.* 19, 149-162 (2008) Zbl 1164.11029 MR 2411018 · Zbl 1164.11029
- [28] Pink, R.: A combination of the conjectures of Mordell-Lang and André-Oort. In: *Geometric Methods in Algebra and Number Theory*, Progr. Math. 235, Birkhäuser, Boston, MA, 251-282 (2005) Zbl 1200.11041 MR 2166087 · Zbl 1200.11041
- [29] Poonen, B.: Gonality of modular curves in characteristic p . *Math. Res. Lett.* 14, 691-701 (2007) Zbl 1138.14016 MR 2335995 · Zbl 1138.14016
- [30] Salehi Golsefidy, A.: Super-approximation, II: the p -adic case and the case of bounded powers of square-free integers. *J. Eur. Math. Soc.* 21, 2163-2232 (2019) Zbl 1422.22013 MR 3959861 · Zbl 1422.22013
- [31] Salehi Golsefidy, A., Varjshu, P. P.: Expansion in perfect groups. *Geom. Funct. Anal.* 22, 1832-1891 (2012) Zbl 1284.20044 MR 3000503 · Zbl 1284.20044
- [32] Tsimerman, J.: The André-Oort conjecture for Ag . *Ann. of Math. (2)* 187, 379-390 (2018) Zbl 1415.11086 MR 3744855 · Zbl 1415.11086
- [33] Ullmo, E.

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.