As Wikipedia informs us, “The Killing form was essentially introduced into Lie algebra theory by É. Car- tan [Sur la structure des groupes de transformations finis et continus. Paris: Nony et Co. (1894; JFM 25.0638.02)] in his thesis. The name “Killing form” first appeared in a paper of Armand Borel in 1951.” Killing forms on Riemannian manifolds are exterior forms whose covariant derivatives with respect to the Levi-Civita connection is totally skew-symmetric. They have been introduced by physicists as first integrals of the equation of motion, Killing forms have had an important role in several classical problems of Lie theory and mathematical physics.

The contents of the paper is best described by its abstract.

“We study left-invariant Killing $k$-forms on simply connected 2-step nilpotent Lie groups endowed with a left-invariant Riemannian metric. For $k = 2, 3$, we show that every left-invariant Killing $k$-form is a sum of Killing forms on the factors of the de Rham decomposition. Moreover, on each irreducible factor, non-zero Killing 2-forms define (after some modification) a bi-invariant orthogonal complex structure and non-zero Killing 3-forms arise only if the Riemannian Lie group is naturally reductive when viewed as a homogeneous space under the action of its isometry group. In both cases, $k = 2$ or $k = 3$, we show that the space of left-invariant Killing $k$-forms of an irreducible Riemannian 2-step nilpotent Lie group is at most one-dimensional.”

Reviewer: Lakehal Belarbi (Mostaganem)


[16] Semmelmann, U., Killing forms on \( \{ (\mathfrak{g} \mathfrak{l}(2, \mathbb{C}), \mathfrak{sp}_n(\mathbb{C}) \} \) and \( \{ (\mathfrak{g} \mathfrak{sl}(m, \mathbb{C}) \) \)-manifolds, J. Geom. Phys., 56, 1752-1766 (2006) · Zbl 1105.53040 · doi:10.1016/j.geomphys.2005.10.003


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