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Summary: Reflected Brownian motion (RBM) in a convex polyhedral cone arises in a variety of applications ranging from the theory of stochastic networks to mathematical finance, and under general stability conditions, it has a unique stationary distribution. In such applications, to implement a stochastic optimization algorithm or quantify robustness of a model, it is useful to characterize the dependence of stationary performance measures on model parameters. In this paper, we characterize parametric sensitivities of the stationary distribution of an RBM in a simple convex polyhedral cone, that is, sensitivities to perturbations of the parameters that define the RBM – namely the covariance matrix, drift vector, and directions of reflection along the boundary of the polyhedral cone. In order to characterize these sensitivities, we study the long-time behavior of the joint process consisting of an RBM along with its so-called derivative process, which characterizes pathwise derivatives of RBMs on finite time intervals. We show that the joint process is positive recurrent and has a unique stationary distribution and that parametric sensitivities of the stationary distribution of an RBM can be expressed in terms of the stationary distribution of the joint process. This can be thought of as establishing an interchange of the differential operator and the limit in time. The analysis of ergodicity of the joint process is significantly more complicated than that of the RBM because of its degeneracy and the fact that the derivative process exhibits jumps that are modulated by the RBM. The proofs of our results rely on path properties of coupled RBMs and contraction properties related to the geometry of the polyhedral cone and directions of reflection along the boundary. Our results are potentially useful for developing efficient numerical algorithms for computing sensitivities of functionals of stationary RBMs.

MSC:
60J65 Brownian motion
60G17 Sample path properties
60G40 Stopping times; optimal stopping problems; gambling theory
65C35 Stochastic particle methods

Keywords:
reflected Brownian motion; stationary distribution; sensitivity analysis; pathwise derivatives; derivative process; Skorokhod reflection problem; derivative problem; asymptotic coupling method

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