Hain, Richard

Hodge theory of the Turaev cobracket and the Kashiwara-Vergne problem. (English)

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Denote the set of free homotopy classes of maps $S^1 \to X$ in a topological space X by $\lambda(X)$ and the free $\mathbb{Q}$-module it generates by $\mathbb{Q}\lambda(X)$. When X is an oriented surface with a nowhere vanishing vector field ξ, there is a map

$$\delta_\xi : \mathbb{Q}\lambda(X) \to \mathbb{Q}\lambda(X) \otimes \mathbb{Q}\lambda(X),$$

called the *Turaev cobracket*, that gives $\mathbb{Q}\lambda(X)$ the structure of a Lie coalgebra. The cobracket was first defined by [*V. G. Turaev*, Mat. Sb., Nov. Ser. 106(148), 566–588 (1978; Zbl 0384.57004)] on $\mathbb{Q}\lambda(M)/\mathbb{Q}$ (with no framing) and lifted to $\mathbb{Q}\lambda(M)$ for framed surfaces in [*V. G. Turaev*, Ann. Sci. Éc. Norm. Supér. (4) 24, No. 6, 635–704 (1991; Zbl 0758.57011)] and [*A. Alekseev* et al., "The Goldman-Turaev Lie bialgebra and the Kashiwara-Vergne problem in higher genera", Preprint, arXiv:1804.09566]. The cobracket δ_ξ and the Goldman bracket [*R. Hain*, "Hodge theory of the Goldman bracket", Preprint, arXiv:1710.06053]

$$\{\ ,\ \} : \mathbb{Q}\lambda(X) \otimes \mathbb{Q}\lambda(X) \to \mathbb{Q}\lambda(X)$$

endow $\mathbb{Q}\lambda(X)$ with the structure of an involutive Lie bialgebra [*V. G. Turaev*, Ann. Sci. Éc. Norm. Supér. (4) 24, No. 6, 635–704 (1991; Zbl 0758.57011); *M. Chas*, Topology 43, No. 3, 543–568 (2004; Zbl 1050.57014); *N. Kawazumi* and *Y. Kuno*, Ann. Inst. Fourier 65, No. 6, 2711–2762 (2015; Zbl 1370.57009)]. The value of the cobracket on a loop $a \in \lambda(X)$ is obtained by representing it by an immersed circle $\alpha : S^1 \to X$ with transverse self intersections and trivial winding number relative to ξ. Each double point P of α divides it into two loops based at P, which we denote by α'_P and α''_P. Let $\epsilon_P = \pm 1$ be the intersection number of the initial arcs of α'_P and α''_P. The cobracket of a is then defined by

$$\delta_\xi(a) = \sum_P \epsilon_P (a'_P \otimes a''_P - a''_P \otimes a'_P), \tag{1}$$

where a'_P and a''_P are the classes of α'_P and α''_P, respectively. The powers of the augmentation ideal I of $\mathbb{Q}\pi_1(X,x)$ define the I-adic topology on it and induce a topology on $\mathbb{Q}\lambda(X)$. *N. Kawazumi* and *Y. Kuno* [Ann. Inst. Fourier 65, No. 6, 2711–2762 (2015; Zbl 1370.57009)] showed that δ_ξ is continuous in the I-adic topology and thus induces a map

$$\delta_\xi : \mathbb{Q}\lambda(X)^\wedge \to \mathbb{Q}\lambda(X)^\wedge \widehat{\otimes} \mathbb{Q}\lambda(X)^\wedge$$

on I-adic completions. This and the completed Goldman bracket give $\mathbb{Q}\lambda(X)^\wedge$ the structure of an involutive completed Lie bialgebra [loc. cit.].

Now suppose that X is a smooth affine curve over $\mathbb{C}$ or, equivalently, the complement of a non-empty finite set D in a compact Riemann surface $\overline{X}$. In this case $\mathbb{Q}\lambda(X)^\wedge$ has a canonical pro-mixed Hodge structure [*R. M. Hain*, K-Theory 1, No. 3, 271–324 (1987; Zbl 0637.55006)]. In particular, it has a *weight filtration*

$$\cdots \subseteq W_{-2}\mathbb{Q}\lambda(X)^\wedge \subseteq W_{-1}\mathbb{Q}\lambda(X)^\wedge \subseteq W_0\mathbb{Q}\lambda(X)^\wedge = \mathbb{Q}\lambda(X)^\wedge$$

and its complexification $\mathbb{C}\lambda(X)^\wedge$ has a *Hodge filtration*

$$\cdots \supset F^{-2}\mathbb{C}\lambda(X)^\wedge \supset F^{-1}\mathbb{C}\lambda(X)^\wedge \supset F^0\mathbb{C}\lambda(X)^\wedge \supset F^1\mathbb{C}\lambda(X)^\wedge = 0.$$

The Hodge filtration depends on the algebraic structure on X, while the weight filtration is topologically determined and so does not depend on the complex structure. The weight filtration on $\mathbb{Q}\lambda(X)^\wedge$ is the image of the weight filtration of $\mathbb{Q}\pi_1(X,x)^\wedge$, which is determined uniquely by the conditions that $W_{-1}\mathbb{Q}\pi_1(X,x)^\wedge = I$, $W_{-2}\mathbb{Q}\pi_1(X,x)^\wedge = \ker\{I \to H_1(\overline{X})\}$, and by the condition that $W_{-m-2}\mathbb{Q}\pi_1(X,x)^\wedge$ is the ideal generated by $W_{-1}W_{-m-1}$ and $W_{-2}W_{-m}$. This pro-mixed Hodge structure contains subtle geometric and arithmetic information about X. The first main result of the paper is that the Turaev cobracket is compatible with this structure.

Theorem 1. If ξ is a nowhere vanishing holomorphic vector field on X that is meromorphic on $\overline{X}$, then

$$\delta_\xi : \mathbb{Q}\lambda(X)^\wedge \otimes \mathbb{Q}(-1) \to \mathbb{Q}\lambda(X)^\wedge \widehat{\otimes} \mathbb{Q}\lambda(X)^\wedge$$

is a morphism of pro-mixed Hodge structures, so that $\mathbb{Q}\lambda(X)^\wedge \otimes \mathbb{Q}(1)$ is a complete Lie coalgebra in the category of pro-mixed Hodge structures.

We call such a framing ξ an *algebraic framing*. The main result of [*R. Hain*, Geom. Topol. 24, No. 4, 1841–1906 (2020; Zbl 1470.14017)] asserts that

$$\{\ ,\ \} : \mathbb{Q}\lambda(X)^\wedge \otimes \mathbb{Q}\lambda(X)^\wedge \to \mathbb{Q}\lambda(X)^\wedge \otimes \mathbb{Q}(1)$$

is a morphism of mixed Hodge structure (MHS), so that $\mathbb{Q}\lambda(X)^\wedge \otimes \mathbb{Q}(-1)$ is a complete Lie algebra in the category of pro-mixed Hodge structures.

Corollary 1. If ξ is a quasi-algebraic framing of X, then $\big(\mathbb{Q}\lambda(X)^\wedge, \{\ ,\ \}, \delta_\xi\big)$ is a "twisted" completed Lie bialgebra in the category of pro-mixed Hodge structures.

By "twisted" one means that one has to twist both the bracket and cobracket by $\mathbb{Q}(\pm 1)$ to make them morphisms of MHS. There is no one twist of $\mathbb{Q}\lambda(X)$ that makes them simultaneously morphisms of MHS. Let $\vec{\mathsf{v}}$ be a non-zero tangent vector of $\overline{X}$ at a point of D. Standard results in Hodge theory (see [*R. Hain*, Geom. Topol. 24, No. 4, 1841–1906 (2020; Zbl 1470.14017)]) imply:

Corollary 2. Hodge theory determines torsors of compatible isomorphisms

$$\big(\mathbb{Q}\lambda(X)^\wedge, \{\ ,\ \}, \delta_\xi\big) \xrightarrow{\simeq} \Big(\prod_{m\geq 0} \mathrm{Gr}^W_{-m}\,\mathbb{Q}\lambda(X)^\wedge, \mathrm{Gr}^W_\bullet\{\ ,\ \}, \mathrm{Gr}^W_\bullet\,\delta_\xi\Big) \tag{2}$$

of the Goldman-Turaev Lie bialgebra with the associated weight graded Lie bialgebra and of the completed Hopf algebras

$$\mathbb{Q}\pi_1(X,\vec{\mathsf{v}})^\wedge \xrightarrow{\simeq} \prod_{m\geq 0} \mathrm{Gr}^W_{-m}\,\mathbb{Q}\pi_1(X,\vec{\mathsf{v}})^\wedge \tag{3}$$

under which the logarithm of the boundary circle lies in $\mathrm{Gr}^W_{-2}\,\mathbb{Q}\pi_1(X,\vec{\mathsf{v}})^\wedge$. These isomorphisms are torsors under the prounipotent radical $U^{\mathrm{MT}}_{X,\vec{\mathsf{v}}}$ of the Mumford-Tate group of the MHS on $\mathbb{Q}\pi_1(X,\vec{\mathsf{v}})^\wedge$.

Let $\overline{S}$ be a closed oriented surface of genus g and $P = \{x_0,\dots,x_n\}$ a finite subset. Set $S = \overline{S} - P$. Assume that S is hyperbolic; that is, $2g-1+n>0$. Suppose that ξ_o is a framing of S. Denote the index (or local degree) of ξ_o at x_j by d_j. Let $\mathbf{d} = (d_0,\dots,d_n) \in \mathbb{Z}^{n+1}$ be the vector of local degrees of ξ_o. The Poincaré-Hopf Theorem implies that $\sum d_j = 2-2g$. Also denote the category of mixed Tate motives unramified over $\mathbb{Z}$ by $\mathsf{MTM}(\mathbb{Z})$. Denote the prounipotent radical of its tannakian fundamental group $\pi_1(\mathsf{MTM},\omega^B)$ (with respect to the Betti realization ω^B) by $\mathcal{K}$. Denote the relative completion of the mapping class group of $(\overline{S},P,\vec{\mathsf{v}}_o)$ by $\mathcal{G}_{g,n+u}$ and its prounipotent radical by $\mathcal{U}_{g,n+u}$. (See [*R. Hain*, J. Am. Math. Soc. 10, No. 3, 597–651 (1997; Zbl 0915.57001)] for definitions.) These act on $\mathbb{Q}\pi_1(S,\vec{\mathsf{v}}_o)^\wedge$. Denote the image of $\mathcal{U}_{g,n+u}$ in $\operatorname{Aut}\mathbb{Q}\pi_1(S,\vec{\mathsf{v}}_o)^\wedge$ by $\mathbb{U}_{g,n+u}$. The vector field ξ_o determines a homomorphism $\mathbb{U}_{g,n+u} \to H_1(\overline{S})$ that depends only on the vector $\mathbf{d}$ of local degrees of ξ. Denote its kernel by $\overline{\mathbb{U}}^{\mathbf{d}}_{g,n+u}$. *Y. Ihara* and *H. Nakamura* [J. Reine Angew. Math. 487, 125–151 (1997; Zbl 0910.14010)] construct canonical smoothings of each maximally degenerate stable curve X_0 of type $(g,n+1)$ over $\mathbb{Z}[[q_1,\dots,q_N]]$ for all $n\geq 0$, where $N = \dim \mathbb{M}_{g,n+1}$. Associated to each tangent vector $\vec{\mathsf{v}} = \pm\partial/\partial q_j$ of $\overline{\mathbb{M}}_{g,n+1}$ at the point corresponding to X_0, there is a limit pro-MHS on $\mathbb{Q}\lambda(X)^\wedge$, that we denote by $\mathbb{Q}\lambda(X_{\vec{\mathsf{v}}})^\wedge$.

Hypothesis. The limit MHS on $\mathbb{Q}\lambda(X_{\vec{\mathsf{v}}})^\wedge$ is the Hodge realization of a pro-object of $\mathsf{MTM}(\mathbb{Z})$. Equivalently, the Mumford-Tate group of the MHS on $\mathbb{Q}\lambda(X_{\vec{\mathsf{v}}})^\wedge$ is isomorphic to $\pi_1(\mathsf{MTM},\omega^B)$.

Theorem 4. If $2g+n>1$ (i.e., S is hyperbolic), then the group $\widehat{\mathbb{U}}^{\mathbf{d}}_{g,n+u}$ does not depend on the choice of a quasi-algebraic structure $\phi : (\overline{S},P,\vec{\mathsf{v}}_o,\xi_o) \to (\overline{X},D,\vec{\mathsf{v}},\xi)$. The group $\overline{\mathbb{U}}^{\mathbf{d}}_{g,n+u}$ is normal in $\widehat{\mathbb{U}}^{\mathbf{d}}_{g,n+u}$. If we assume Hypothesis 1, there is a canonical surjective group homomorphism $\mathcal{K} \to \widehat{\mathbb{U}}^{\mathbf{d}}_{g,n+u}/\overline{\mathbb{U}}^{\mathbf{d}}_{g,n+u}$, where $\mathcal{K}$ denotes the prounipotent radical of $\pi_1(\mathsf{MTM})$.

From [*R. Hain*, J. Am. Math. Soc. 10, No. 3, 597–651 (1997; Zbl 0915.57001)] that the completion of

$\Gamma_{g,m+\vec{r}}$ relative to $\rho:\Gamma_{g,m+\vec{r}}\to \mathrm{Sp}(H_{\mathbb{Q}})$ is an affine $\mathbb{Q}$-group $\mathcal{G}_{g,m+\vec{r}}$ that is an extension

$$1\to \mathcal{U}_{g,m+\vec{r}}\to \mathcal{G}_{g,m+\vec{r}}\to \mathrm{Sp}(H)\to 1$$

of affine $\mathbb{Q}$-groups, where $\mathcal{U}_{g,m+\vec{r}}$ is prounipotent. There is a Zariski dense homomorphism $\tilde{\rho}:\Gamma_{g,m+\vec{r}}\to \mathcal{G}_{g,m+\vec{r}}(\mathbb{Q})$ whose composition with the homomorphism $\mathcal{G}_{g,m+\vec{r}}(\mathbb{Q})\to \mathrm{Sp}(H_{\mathbb{Q}})$ is ρ. When $g=0$, $\mathrm{Sp}(H)$ is trivial and $\mathcal{G}_{0,m+\vec{r}}$ is the unipotent completion $\Gamma^{\mathrm{un}}_{0,m+\vec{r}}$. The action of the mapping class group $\Gamma_{g,n+u}$ on $\mathbb{Q}\pi_1(S,\vec{\mathsf{v}}_o)$ induces an action on $\mathbb{Q}\lambda(S)$ which preserves the Goldman bracket. The stabilizer of ξ_o preserves the Turaev cobracket. The universal mapping property of relative completion implies that $\mathcal{G}_{g,n+u}$ acts on $\mathbb{Q}\pi_1(S,\vec{\mathsf{v}}_o)^\wedge$ and $\mathbb{Q}\lambda(S)^\wedge$. Since the image of the mapping class group in $\mathcal{G}_{g,n+u}$ is Zariski dense, this action preserves the Goldman bracket. A quasi-complex structure

$$\phi:(\overline{S},P,\vec{\mathsf{v}}_o,\xi_o)\to(\overline{X},D,\vec{\mathsf{v}},\xi)$$

on $(\overline{S},P,\vec{\mathsf{v}}_o,\xi_o)$ determines an isomorphism $\Gamma_{g,n+u}\cong\pi_1(\mathbb{M}_{g,n+u},\phi_o)$. The corresponding MHS on the relative completion $\mathcal{G}_{g,n+u}$ corresponds to an action of $\pi_1(\mathsf{MHS})$ on $\mathcal{G}_{g,n+u}$. The quasi-complex structure ϕ determines a semi-direct product

$$\pi_1(\mathsf{MHS})\ltimes\mathcal{G}_{g,n+u}.$$

Since the natural homomorphism $\mathcal{G}_{g,n+u}\to \mathrm{Aut}\,\mathbb{Q}\pi_1(X,\vec{\mathsf{v}}_o)^\wedge$ is a morphism of MHS [*R. Hain*, J. Am. Math. Soc. 10, No. 3, 597–651 (1997; Zbl 0915.57001)], the monodromy homomorphism extends to a homomorphism

$$\pi_1(\mathsf{MHS})\ltimes\mathcal{G}_{g,n+u}\to \mathrm{Aut}\,\mathbb{Q}\pi_1(X,\vec{\mathsf{v}}_o)^\wedge.$$

Denote its image by $\widehat{\mathcal{G}}_{g,n+u}$ and the image of $\mathcal{G}_{g,n+u}$ by $\overline{\mathcal{G}}_{g,n+u}$. It is normal in $\widehat{\mathcal{G}}_{g,n+u}$. The group $\widehat{\mathcal{G}}_{g,n+u}$ is an extension

$$1\to\widehat{\mathbb{U}}_{g,n+u}\to\widehat{\mathcal{G}}_{g,n+u}\to \mathrm{GSp}(H)\to 1,$$

where GSp denotes the general symplectic group and $\widehat{\mathbb{U}}_{g,n+u}$ is prounipotent. One can argue as in [*R. Hain* and *M. Matsumoto*, J. Inst. Math. Jussieu 4, No. 3, 363–403 (2005; Zbl 1094.14013)] that, if $g\geq 3$, then then $\mathcal{U}^{\mathrm{MT}}_{X,\vec{\mathsf{v}}}\to\widehat{\mathbb{U}}_{g,n+u}$ is an isomorphism if and only if $\pi_1(\mathsf{MHS})\to \mathrm{GSp}(H)$ is surjective; the Griffiths invariant $\nu(\overline{X})\in \mathrm{Ext}^1_{\mathsf{MHS}}(\mathbb{Q},PH^3(\mathrm{Jac}\,\overline{X}(2)))$ of the Ceresa cycle in $\mathrm{Jac}\,\overline{X}$ is non-zero; and if the points $\kappa_j:=(2g-2)x_j-K_{\overline{X}}\in(\mathrm{Jac}\,\overline{X})\otimes\mathbb{Q}$, $0\leq j\leq n$, are linearly independent over $\mathbb{Q}$. This holds for general $(\overline{X},D,\vec{\mathsf{v}})$.

Proposition. For each complex structure $\phi:(\overline{S},P,\vec{\mathsf{v}}_o)\to(\overline{X},D,\vec{\mathsf{v}})$, the coordinate ring $\mathcal{O}(\widehat{\mathcal{G}}_{g,n+u}/\overline{\mathcal{G}}_{g,n+u})$ has a canonical MHS. These form an admissible variation of MHS over $\mathbb{M}_{g,n+u}$ with trivial monodromy. Consequently, the MHS on $\mathcal{O}(\widehat{\mathbb{U}}_{g,n+u}/\overline{\mathbb{U}}_{g,n+u})$ does not depend on the complex structure ϕ.

Reviewer: Mohammad Reza Rahmati (León)

MSC:

14C30 Transcendental methods, Hodge theory (algebro-geometric aspects)
14H10 Families, moduli of curves (algebraic)
58A12 de Rham theory in global analysis
17B62 Lie bialgebras; Lie coalgebras
57K20 2-dimensional topology (including mapping class groups of surfaces, Teichmüller theory, curve complexes, etc.)

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Turaev cobracket; Goldman bracket; Lie bialgebra; Hodge theory

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