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The article investigates the differential equation

\[ \varepsilon^2 u_{xx} = u^3 - \lambda u + \delta g(x) \]

with Neumann boundary conditions on the interval \( x \in [0, 2\pi] \), where \( \varepsilon \in [0, 1] \), \( \lambda \in [0, 4] \) and \( \delta \in [0, 1] \) are parameters and \( g(x) \) is chosen as \( \cos(x) \). The equation corresponds to the Duffing oscillation with a softening nonlinearity.

Several aspects of the system are considered: For the algebraic equation with \( \varepsilon = 0 \) and \( \delta \) fixed regular solutions exist only for sufficiently large values of \( \lambda \). For the singular limit \( 0 < \varepsilon \ll 1 \) families of singular asymptotic solutions with interior solution layers are observed.

The major part of the article is concerned with the bifurcation of solutions from the trivial state at \( \lambda = \varepsilon^2 n^2 \) with \( n \in \mathbb{N} \) for the autonomous system with \( \delta = 0 \) and their behaviour for increasing values of \( \delta \). It turns out, that depending on the parity of \( n \) the observed bifurcations are symmetric of pitchfork type or cusp catastrophes, which lead to disconnected solution branches. Also the solution behaviour for the limiting cases \( \varepsilon^2/\lambda \to 0 \) and \( \varepsilon^2/\lambda \to \infty \) is investigated.

Also the applied numerical procedure to locate the solution branches is explained in some detail.

Since the authors observed a lot of bifurcation scenarios, it would be interesting to find some kind of organizing center by following e.g. branches of cusp points and finding higher order catastrophes.

Reviewer: Alois Steindl (Wien)

MSC:

34C23 Bifurcation theory for ordinary differential equations
34E10 Perturbations, asymptotics of solutions to ordinary differential equations
34E15 Singular perturbations for ordinary differential equations
34B08 Parameter dependent boundary value problems for ordinary differential equations
37C60 Nonautonomous smooth dynamical systems

Keywords:

bifurcation; Neumann boundary conditions; layer solutions; self-focusing solutions; Duffing equation; tearing

Software:

Chebfun; Matlab; ode113; ode45; ode23s; Ode15s; ode23; MATLAB ODE suite

Full Text: DOI

References:


[20] Seydel, R., Practical Bifurcation and Stability Analysis (2010), Springer Verlag · Zbl 1195.34004


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