Some sum-product estimates in matrix rings over finite fields. (English) 

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Summary: We study some sum-product problems over matrix rings. Firstly, for $A, B, C \subseteq M_n(\mathbb{F}_q)$, we have
$$|A + BC| \gtrsim q^{n^2},$$
whenever $|A||B||C| \gtrsim q^{3n^2 - \frac{n+1}{2}}$. Secondly, if a set $A$ in $M_n(\mathbb{F}_q)$ satisfies $|A| \geq C(n)q^{n^2-1}$ for some sufficiently large $C(n)$, then we have
$$\max\{|A + A|, |AA|\} \gtrsim \min\left\{\frac{|A|^2}{q^{n^2 - \frac{n+1}{2}}}, q^{n^2/3}|A|^{2/3}\right\}.$$

These improve the results due to The and Vinh (2020), and generalize the results due to Mohammadi, Pham, and Wang (2021). We also give a new proof for a recent result due to The and Vinh (2020). Our method is based on spectral graph theory and linear algebra.

MSC:

11B75 Other combinatorial number theory
11B30 Arithmetic combinatorics; higher degree uniformity
68R05 Combinatorics in computer science

Keywords:
sum-product estimates; spectral graph theory; finite field

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References:

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