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On space of holomorphic functions with boundary smoothness and its dual. (Russian. English summary) Zbl 1487.32009

Ufim. Mat. Zh. 13, No. 3, 82-96 (2021); translation in Ufa Math. J. 13, No. 3, 80-94 (2021).

Summary: We consider a Fréchet-Schwartz space $A_{\mathcal{H}}(\Omega)$ of functions holomorphic in a bounded convex domain Ω in a multidimensional complex space and smooth up to the boundary with the topology defined by means of a countable family of norms. These norms are constructed via some family $\mathcal{H}$ of convex separately radial weight functions in $\mathbb{R}^n$. We study the problem on describing a strong dual space for this space in terms of the Laplace transforms of functionals. An interest to such problem is motivated by the researches by B. A. Derzhavets devoted to classical problems of theory of linear differential operators with constant coefficients and the researches by A. V. Abanin, S. V. Petrov and K. P. Isaev of modern problems of the theory of absolutely representing systems in various spaces of holomorphic functions with given boundary smoothness in convex domains in complex space; these problems were solved by Paley-Wiener-Schwartz type theorems. Our main result states that the Laplace transform is an isomorphism between the strong dual of our functional space and some space of entire functions of exponential type in $\mathbb{C}^n$, which is an inductive limit of weighted Banach spaces of entire functions. This result generalizes the corresponding result of the second author in [Vladikavkaz. Mat. Zh. 22, No. 3, 100–111 (2020; Zbl 1474.32003)]. To prove this theorem, we apply the scheme proposed by M. Neymark and B. A. Taylor. On the base of results from monograph by *L. Hörmander* [An introduction to complex analysis in several variables. 3rd revised ed. Amsterdam etc.: North-Holland (1990; Zbl 0685.32001)], a problem of solvability of systems of partial differential equations in $A^m_{\mathcal{H}}(\Omega)$ is considered. An analogue of a similar result from monograph by L. Hörmander is obtained. In this case we employ essentially the properties of the Young-Fenchel transform of functions in the family $\mathcal{H}$.

MSC:

32A15 Entire functions of several complex variables
32A40 Boundary behavior of holomorphic functions of several complex variables

Keywords:

convex domain in $\mathbb{C}^n$; holomorphic functions smooth up to the boundary; Laplace transform; entire functions

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References:

[1] R. S. Yulmukhametov, "Space of analytic functions with prescribed growth near the boundary", Math. Notes, 32:1 (1982), 499-508 · Zbl 0502.46012 · doi:10.1007/BF01137223

[2] B. A. Derzhavets, Differential operators with constant coefficients in spaces of analytic functions of many variables, PhD thesis, Univ. of Rostov-on-Don, 1983 (in Russian)

[3] E. M. Dyn'kin, "Pseudoanalytic continuation of smooth functions. Uniform scale", Proc. Seventh Winter School "Mathematical programming and related issues" (Drogobych, 1974), M., 1976, 40-74 (in Russian)

[4] R. S. Yulmukhametov, "Quasianalytical classes of functions in convex domains", Math. USSR-Sb., 58:2 (1987), 505-523 · Zbl 0625.30037 · doi:10.1070/SM1987v058n02ABEH003117

[5] I. Kh. Musin, "Spaces of functions holomorphic in convex bounded domains of and smooth up to the boundary", Advances in Mathematics Research, Nova Science Publishers, New York, 2002, 63-74

[6] K. V. Trounov, R. S. Yulmukhametov, "Quasi-analytic Carleman classes on bounded domains", St. Petersburg Math. J., 20:2 (2009), 289-317 · Zbl 1206.30049 · doi:10.1090/S1061-0022-09-01048-6

[7] R. A. Gaisin, "A universal criterion for quasi-analytic classes in Jordan domains", Sb. Math., 209:12 (2018), 1728-1744 · Zbl 1411.30024 · doi:10.1070/SM8655

[8] R. A. Gaisin, "Quasi-analyticity criteria of Salinas-Korenblum type for general domains", Ufa Math. J., 5:3 (2013), 28-39· doi:10.13108/2013-5-3-28

[9] A. V. Abanin, T. M. Andreeva, "Analytic description of the spaces dual to spaces of holomorphic functions of given growth

on Carathéodory domains", Math. Notes, 104:3 (2018), 321-330 · Zbl 1411.30042 · doi:10.1134/S0001434618090018

[10] A. V. Abanina, S. V. Petrov, "Minimal absolutely representing systems of exponential functions in spaces of analytic functions with given boundary smoothness", Vladikavkaz. Matem. Zhurn., 14:3 (2012), 13-30 (in Russian) · Zbl 1326.30005

[11] S. V. Petrov, "Existence of absolutely representing exponential systems in spaces of analytic functions", Izv. VUZov. Sever. Kavkaz. Region. Estestv. Nauki, 5 (2010), 25-31 (in Russian) · Zbl 1224.46047

[12] K. P. Isaev, "Representing systems of exponentials in projective limits of weighted subspaces of \(A^\infty(D)\)", Russian Math., 63:1 (2019), 24-34 · Zbl 1439.30085 · doi:10.3103/S1066369X19010031

[13] I. Kh. Musin, "On a space of holomorphic functions on a bounded convex domain of \(\mathbb{C}^n\) and smooth up to the boundary and its dual space", Vladikavkaz. Mat. Zh., 22:3 (2020), 100-111 · Zbl 1474.32003

[14] M. Neymark, "On the Laplace transform of functionals on classes of infinitely differentiable functions", Ark. math., 7 (1969), 577-594 · Zbl 0172.42101 · doi:10.1007/BF02590896

[15] B. A. Taylor, "Analytically uniform spaces of infinitely differentiable functions", Commun. on pure and appl. mathematics, 24:1 (1971), 39-51 · Zbl 0205.41501 · doi:10.1002/cpa.3160240105

[16] Il'dar Kh. Musin, Polina V. Yakovleva, "On a space of smooth functions on a convex unbounded set in admitting holomorphic extension in \(\mathbb{C}^n\)", Central European Journal of Mathematics, 10:2 (2012), 665-692 · Zbl 1252.46025 · doi:10.2478/s11533-011-0142-8

[17] J. Sebastião e Silva, "Su certe classi di spazi localmente convessi importanti per le applicazioni", Rend. Mat. Appl., 14 (1955), 388-410 · Zbl 0064.35801

[18] V. V. Zharinov, "Compact families of locally convex topological vector spaces, Fréchet-Schwartz and dual Fréchet-Schwartz spaces", Russ. Math. Surv., 34:4 (1979), 105-143 · Zbl 0443.46002 · doi:10.1070/RM1979v034n04ABEH002963

[19] H. Komatsu, "Projective and injective limits of weakly compact sequences of locally convex spaces", J. Math. Soc. Japan, 19:3 (1967), 366-383 · Zbl 0168.10603 · doi:10.2969/jmsj/01930366

[20] A. Grothendieck, Produits tensoriels topologiques et espaces nucléaires, Mem. Amer. Math. Soc., 16, Amer. Math. Soc., Providence, RI, 1955 · Zbl 0064.35501

[21] V. V. Napalkov, Convolution equations in multi-dimensional spaces, Nauka, M., 1982 (in Russian)

[22] R. Meise, D. Vogt, Introduction to Functional Analysis, Clarendon, Oxford, 1997 · Zbl 0924.46002

[23] L. Hörmander, An Introduction to Complex Analysis in Several Variables, North Holland, 3rd edition, Amsterdam, 1990 · Zbl 0685.32001

[24] B. A. Derzhavets, "Systems of partial differential equations in a space of functions analytic in the ball and having a given growth near its boundary", Siberian Math. J., 26:2 (1985), 231-236 · Zbl 0595.35101 · doi:10.1007/BF00968765