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On the isoperimetric inequality for the magnetic Robin Laplacian with negative boundary parameter. (English) Zbl 1487.35260

J. Geom. Anal. 32, No. 6, Paper No. 182, 20 p. (2022); correction ibid. 32, No. 9, Paper No. 237, 2 p. (2022).

Summary: We consider the magnetic Robin Laplacian with a negative boundary parameter on a bounded, planar C^2-smooth domain. The respective magnetic field is homogeneous. Among a certain class of domains, we prove that the disk maximises the ground-state energy under the fixed perimeter constraint provided that the magnetic field is of moderate strength. This class of domains includes, in particular, all domains that are contained upon translations in the disk of the same perimeter and all centrally symmetric domains.

MSC:

35P15	Estimates of eigenvalues in context of PDEs
35J25	Boundary value problems for second-order elliptic equations
81Q10	Selfadjoint operator theory in quantum theory, including spectral analysis

Keywords:

magnetic Robin Laplacian; homogeneous magnetic field; lowest eigenvalue; isoperimetric inequality; parallel coordinates; centrally symmetric domain

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