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Universality of Wolfram’s 2, 3 Turing machine. (English) [Zbl 1487.68103]

Complex Syst. 29, No. 1, 1–44 (2020).

This article concerns the question of whether the smallest universal Turing machine has two states and uses an alphabet of three letters, a problem addressed in [S. Wolfram, A new kind of science. Champaign, IL: Wolfram Media (2002; Zbl 1022.68084)]. In [C. E. Shannon, “A universal Turing machine with two internal states”, in: Automata studies. Princeton: Princeton University. 157–165 (1956)] it was proven that two states could suffice, but it was later proven that two states and two letters does not suffice (see [M. Margenstern, Complex Syst. 19, No. 1, 29–43 (2010; Zbl 1217.68096)]). This article concerns the following Turing machine, called “System 0”:

\[
\begin{array}{cccccccc}
0y & 0y & y & y & x & 1 & 0 & y \\
A & B & A & B & A & B \\
X & X & X & X & X & X & X & X \\
\end{array}
\]

The computation starts with the head left of the leftmost square and in State A, and there is no halting state.

A universal Turing machine would be able to emulate any other machine, i.e., for any Turing machine \(M\) with alphabet \(\Sigma\), there would be a string homomorphism \(f : \Sigma^* \rightarrow \{0, 1\}^*\) such that if \(M\) is given an input \(x\), then the computation of \(M\) on \(x\) is emulated by the computation of System 0 on \(f(x)\), i.e., if at some time \(t\), \(M\) has \(y\) on its tape, then at some time \(n_t\) System 0 has \(f(y)\) on its tape, where \(0 = n_0 < n_1 < n_2 < \cdots\). The main result of this article is that for any Turing machine \(M\) and any positive integer \(N\) there is a string homomorphism enabling System 0 to emulate the computation of \(M\) for at least \(N\) steps. The construction consists of a sequence of such emulations, ending with a system that can so emulate a two-color cyclic tag system known to be universal (see [J. Cocke and M. Minsky, J. Assoc. Comput. Mach. 11, 15–20 (1964; Zbl 0149.12405); M. Cook, Complex Syst. 15, No. 1, 1–40 (2004; Zbl 1167.68387)]) for any finite number of steps.

Unfortunately, the exposition is difficult, occasionally unclear or otherwise problematic, and not self-contained. In addition, the tables exhibiting the machines are badly typeset, and readers may prefer the original article [A. Smith, “The universality of Wolfram’s 2,3 Turing machine” (2007), https://www.wolframscience.com/prizes/tm23/TM23Proof.pdf]; worse, some formulas are badly typeset in both versions. There are several points that seem to require clarification or adjustment, and I cannot follow some steps in the proof.

The rest of the article addresses the complexity of the emulations.

Reviewer: Gregory Loren McColm (Tampa)

MSC:

68Q04 Classical models of computation (Turing machines, etc.)
03D10 Turing machines and related notions

Keywords:
cyclic tag system; emulation; universal Turing machine

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