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A note on statistics simulation for geometric probability problems. (Chinese. English summary)


Summary: Let $D$ be a geometric compact domain in $\mathbb{R}^2$ and $X$ be a statistic based on a random coverage process on $D$. Due to the problem complexity there are no analytic formulas for the mean, variance and distribution function of $X$. A statistical simulation can help us find approximation solutions to them. To use a statistical simulation the result is effected by a set of representative points for $D$. If $D$ is not a rectangle, how to choose a suitable set of representative points is a very important issue. In this paper we discuss a real-life case study that is a geometric coverage problem on the unit cycle covered by $m$ random cycles, which had been discussed in the literature. They proposed four methods of generating a set of representative points of $D$ and gave some evaluation on these four methods. In this paper we consider the case of two random cycles and derive the analytic formulas for the coverage area so that we have a benchmark for comparing the four methods. Our conclusions are consistent with that given in the literature. Furthermore, we find that two methods imply biased estimator of the coverage area with a larger variance. The paper also shows that the distribution of the coverage area can be approximated by a $\beta$ distribution.

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60D05 Geometric probability and stochastic geometry

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