Küçükoğlu, İrem; Şimşek, Burçin; Şimşek, Yılmaz
New classes of Catalan-type numbers and polynomials with their applications related to $p$-adic integrals and computational algorithms. (English) Zbl 1493.05020

Summary: The aim of this paper is to construct generating functions for new classes of Catalan-type numbers and polynomials. Using these functions and their functional equations, we give various new identities and relations involving these numbers and polynomials, the Bernoulli numbers and polynomials, the Stirling numbers of the second kind, the Catalan numbers and other classes of special numbers, polynomials and functions. Some infinite series representations, including the Catalan-type numbers and combinatorial numbers, are investigated. Moreover, some recurrence relations and computational algorithms for these numbers and polynomials are provided. By implementing these algorithms in the Python programming language, we illustrate the Catalan-type numbers and polynomials with their plots under the special conditions. We also give some derivative formulas for these polynomials. Applying the Riemann integral, contour integral, Volkenborn (bosonic $p$-adic) integral and fermionic $p$-adic integral to these polynomials, we also derive some integral formulas. With the help of these integral formulas, we give some identities and relations associated with some classes of special numbers and also the Cauchy-type numbers.

MSC:
05A15 Exact enumeration problems, generating functions
05A19 Combinatorial identities, bijective combinatorics
11B37 Recurrences
11B68 Bernoulli and Euler numbers and polynomials
11S80 Other analytic theory (analogues of beta and gamma functions, $p$-adic integration, etc.)
30E20 Integration, integrals of Cauchy type, integral representations of analytic functions in the complex plane
34A99 General theory for ordinary differential equations
35A99 General topics in partial differential equations
68W01 General topics in the theory of algorithms

Keywords:
generating function; Bernoulli polynomials; Stirling numbers; Catalan numbers; partial differential equations; computational algorithms; $p$-adic integral

Full Text: DOI

References: