This paper deals with the tropical enumerative geometry and its relation to the enumeration of complex and real algebraic curves, notably, with refined tropical enumerative invariants, which depend on a parameter and interpolate between the complex and real curve counting. The most interesting example of a refined invariant was suggested by F. Block and L. Göttsche [Compos. Math. 152, No. 1, 115–151 (2016; Zbl 1348.14125)]. The Block-Göttsche invariant enumerates plane tropical curves of any given degree and genus. The problem addressed in the reviewed paper is to extend the refined count to the case of tropical curves in Euclidean spaces of arbitrary dimension. In the planar case the complex weight of a tropical curve (i.e., the number of complex curves tropicalizing to the given tropical curve) splits into the product of weight of vertices, and the refinement is applied to each vertex separately. However, T. Nishinou and B. Siebert [Duke Math. J. 135, No. 1, 1–51 (2006; Zbl 1105.14073)] showed that the count of spatial tropical curves leads in general to weight depending on the entire tropical curve. The main author’s result designates situations when spatial tropical curves still can be invariantly counted with refined weights defined as products of Block-Göttsche type factors corresponding to the vertices of tropical curves. The most interesting case is the enumeration of rational spatial curves of a given degree whose all but one unbounded edges lie in specific hyperplanes depending on an a priori given 2-form $\omega$, and one unbounded edge lies in a fixed 2-plane. Under the generic choice of such constraints, all tropical curves in count appear to be trivalent, and their refined weights are products of contributions of the vertices. It is proved that the sum of such weights over all suitable tropical curves does not depend on the choice of constraints (but depends on the choice of $\omega$). This can be viewed as a generalization of the enumerative problem considered by G. Mikhalkin [Acta Math. 219, No. 1, 135–180 (2017; Zbl 1468.14092)]. The author also discusses closely related questions like computation of complex Nishinou-Siebert weights of spatial tropical curves in general situation and imposing other types of constraints that lead to invariant refined count of spatial tropical curves.

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MSC:
14N10 Enumerative problems (combinatorial problems) in algebraic geometry
14M25 Toric varieties, Newton polyhedra, Okounkov bodies
14T90 Applications of tropical geometry
14H99 Curves in algebraic geometry

Keywords:
enumerative geometry; tropical curves; refined enumerative invariants; Block-Göttsche polynomial; Nishinou-Siebert multiplicity

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