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A new constraint qualification and sharp optimality conditions for nonsmooth mathematical programming problems in terms of quasidifferentials. (English) Zbl 1493.90219

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The author presents an analysis of a new constraint qualification and a derivation of the strongest existing optimality conditions for nonsmooth mathematical problems with equality and inequality constraints in terms of quasidifferentials. He obtains a convenient description of convex subcones of the contingent cone to a set defined by quasidifferentiable equality and inequality constraints with the use of the new constraint qualification. The author uses this description to obtain the strongest existing necessary optimality conditions for nonsmooth mathematical programming problems in terms of quasidifferentials under less restrictive assumptions than in previous studies. Also, the author presents two examples demonstrating that optimality conditions in terms of quasidifferentials are sometimes better than optimality conditions in terms of various subdifferentials, since they are able to detect the nonoptimality of a given point when optimality conditions based on various subdifferentials fail to do so. Finally, the author presents a comparison between assumptions and constraint qualification used in this paper and those used in previous studies.

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