Carneiro, Emanuel; Chandee, Vorrapan; Chirre, Andrés; Milinovich, Micah B.
On Montgomery’s pair correlation conjecture: a tale of three integrals. (English)

The authors study three integrals related to the celebrated pair correlation conjecture of H. L. Montgomery. The first is the integral of Montgomery’s function $F(\alpha, T)$ in bounded intervals:

$$F(\alpha, T) = \frac{2\pi}{T \log T} \sum_{0 < \gamma, \gamma' \leq T} T^\alpha (\gamma - \gamma') \omega(\gamma - \gamma'),$$

where $\omega(u) = \frac{4}{\pi + u^2}$.

The second is an integral $J(\beta, T)$ introduced by Selberg related to estimating the variance of primes in short intervals:

$$J(\beta, T) = \int_1^T \left( \psi\left(x + \frac{x}{T}\right) - \psi(x) - \frac{x}{T}\right)^2 \frac{dx}{x^2},$$

where $\psi(x)$ is the Chebyshev function.

Finally, the last integral is $I(a, T)$ which is the second moment of the logarithmic derivative of the Riemann zeta-function $\zeta(s)$ near the critical line:

$$I(a, T) = \int_{1}^{T} \left| \zeta'\left(\frac{1}{2} + \frac{a}{\log T} + it\right) \right|^2 dt.$$

Assuming the Riemann hypothesis, it is well known that some conjectured asymptotic formulas for any of the mentioned three integrals are equivalent to Montgomery’s pair correlation conjecture regarding the distribution of the zeroes of $\zeta(s)$ on the critical line $\text{Re}(s) = \frac{1}{2}$.

In the paper under review, the authors assume the Riemann hypothesis and substantially improve the known upper and lower bounds for these integrals by introducing new connections to certain extremal problems in Fourier analysis.

Reviewer: Sami Omar (Sukhair)

MSC:

11M06 $\zeta(s)$ and $L(s, \chi)$

11N05 Distribution of primes

Keywords:
Montgomery’s pair correlation conjecture; Riemann hypothesis; zeroes of the Riemann zeta function

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References:


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