Moraga, Alexis; Ponce, Mario
Relating boundary and interior solutions of the cohomological equation for cocycles by isometries of negatively curved spaces. The Livšic case. (English) Zbl 1496.37050
Nonlinearity 35, No. 4, 1634-1651 (2022).

A cocycle by isometries is a pair \((T, A)\) where \(T : \Omega \to \Omega\) is a self-homeomorphism of a compact metric space and \(A : \Omega \to \text{Isom}(H)\) is a continuous map to the topological group of isometries of another metric space \(H\). Such a cocycle is called reducible if the cohomological equation

\[ A(\omega) \equiv B(T\omega) \cdot [B(\omega)]^{-1} \]

admits a continuous solution \(B : \Omega \to \text{Isom}(H)\).

The main theorem of the paper is a reducibility criterion. The authors assume that \(T\) is a hyperbolic homeomorphism and that \(A\) satisfies a Hölder continuity condition. They also assume that the space \(H\) is Gromov hyperbolic, and that it is uniquely visible, in the sense that every pair of boundary points can be connected by a unique geodesic. In this setting, the authors obtain a sufficient and necessary condition for reducibility, which is formulated in terms of the induced action on the boundary \(\partial H\). Namely, the space \(\Omega \times \partial H\) must be saturated by (regular enough) sections that are invariant under the corresponding skew-product dynamics.

The classical Livšic theorem is an ingredient in the construction of the solution \(B\); it is applied to an \(\mathbb{R}\)-valued cocycle arising from the action on the space of horospheres.

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MSC:
- 37H05 General theory of random and stochastic dynamical systems
- 37H15 Random dynamical systems aspects of multiplicative ergodic theory, Lyapunov exponents
- 37A20 Algebraic ergodic theory, cocycles, orbit equivalence, ergodic equivalence relations
- 37D40 Dynamical systems of geometric origin and hyperbolicity (geodesic and horocycle flows, etc.)

Keywords:
Livšic problem; cocycles by isometries; skew-product dynamics

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