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A gentle introduction to homological mirror symmetry. (English) Zbl 1497.14001

Mirror symmetry, first observed by physicists studying string theory, posits that Calabi-Yau manifolds exist in pairs $(X, X^\vee)$, where the symplectic geometry of $X$ controls the complex geometry of $X^\vee$ and vice-versa. This phenomenon has received a great deal of attention from mathematicians over the last thirty years, and has been given a compelling categorical formulation by Kontsevich known as homological mirror symmetry [M. Kontsevich, “Homological algebra of mirror symmetry”, Preprint, arXiv:alg-geom/9411018]. Although still conjectural, homological mirror symmetry serves as an important organising principle in the field, and it provides a rigorous mathematical formulation of what it means for a complex manifold $X^\vee$ to be mirror to a symplectic manifold $X$. Roughly put, homological mirror symmetry states:

Let $X$ be a symplectic manifold. A complex algebraic manifold $X^\vee$, is said to be mirror dual to $X$ if the bounded derived category of coherent sheaves on $X^\vee$ is equivalent to the bounded derived category constructed the Fukaya category of $X$.

Describing the Fukaya category as well as the bounded derived category of coherent sheaves on $X^\vee$ is equivalent to the bounded derived category constructed the Fukaya category of $X$. The book under review provides an introduction to homological mirror symmetry which is accessible to graduate students in mathematics. In particular, it includes a great amount of background material, and motivational sections. Among the reason it is so approachable is the style it is written: it first provides an introduction to the general ideas surrounding homological mirror symmetry, and then makes the non-standard choice to focus on the symplectic geometry of surfaces and the representation-theory of quivers. While not being the first historical examples of homological mirror symmetry, these topics are technically the most accessible ones, and the book thus provides an excellent starting point for beginners to the topic.

The first part of the book consists of algebraic preliminaries on categories, cohomology, $A_\infty$-categories and quivers. The second part of the book is an introduction to mirror symmetry. It includes an overview of the physics motivation, some basics of symplectic geometry and Fukaya categories, as well as basics of algebraic geometry and derived categories, and a brief description of the role of toric and tropical geometry through the Batyrev-Borisov construction and the SYZ conjecture. While each of these more advanced topics are only briefly touched upon, references are systematically provided for the reader wishing to study further in those directions.

The third part is the heart of the book. The focus is on the symplectic geometry of surfaces and its interaction through homological mirror symmetry with the topic of gentle algebras and quiver representations. Very recent developments in this field, such as the relation between Bridgeland stability conditions on Fukaya categories of surfaces and the geometry of quadratic differentials, are covered.

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MSC:

14–01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to algebraic geometry
14F08 Derived categories of sheaves, dg categories, and related constructions in algebraic geometry
14J33 Mirror symmetry (algebra-geometric aspects)
53D37 Symplectic aspects of mirror symmetry, homological mirror symmetry, and Fukaya category
Keywords:
homological mirror symmetry; quivers

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