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Oscillation of deviating differential equations. (English) Zbl 1499.34344

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Summary: Consider the first-order linear delay (advanced) differential equation

$$x'(t) + p(t)x(\tau(t)) = 0 \quad (x'(t) - q(t)x(\sigma(t)) = 0), \quad t \geq t_0,$$

where p (q) is a continuous function of nonnegative real numbers and the argument $\tau(t)$ ($\sigma(t)$) is not necessarily monotone. Based on an iterative technique, a new oscillation criterion is established when the well-known conditions

$$\limsup_{t\to\infty} \int_{\tau(t)}^{t} p(s)ds > 1 \quad \left(\limsup_{t\to\infty} \int_{t}^{\sigma(t)} q(s)ds > 1\right)$$

and

$$\liminf_{t\to\infty} \int_{\tau(t)}^{t} p(s)ds > \frac{1}{\mathrm{e}} \quad \left(\liminf_{t\to\infty} \int_{t}^{\sigma(t)} q(s)ds > \frac{1}{\mathrm{e}}\right)$$

are not satisfied. An example, numerically solved in MATLAB, is also given to illustrate the applicability and strength of the obtained condition over known ones.

MSC:

34K11 Oscillation theory of functional-differential equations
34K06 Linear functional-differential equations

Keywords:

differential equation; non-monotone argument; oscillatory solution; nonoscillatory solution; Gronwall inequality

Software:

Matlab

Full Text: DOI

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