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Computability and Beltrami fields in Euclidean space. (English. French summary)

Summary: In this article, we pursue our investigation of the connections between the theory of computation and hydrodynamics. We prove the existence of stationary solutions of the Euler equations in Euclidean space, of Beltrami type, that can simulate a universal Turing machine. In particular, these solutions possess undecidable trajectories. Heretofore, the known Turing complete constructions of steady Euler flows in dimension 3 or higher were not associated to a prescribed metric. Our solutions do not have finite energy, and their construction makes crucial use of the non-compactness of $\mathbb{R}^3$, however they can be employed to show that an arbitrary tape-bounded Turing machine can be robustly simulated by a Beltrami flow on $\mathbb{T}^3$ (with the standard flat metric). This shows that there exist steady solutions to the Euler equations on the flat torus exhibiting dynamical phenomena of (robust) computational complexity as high as desired. We also quantify the energetic cost for a Beltrami field on $\mathbb{T}^3$ to simulate a tape-bounded Turing machine, thus providing additional support for the space-bounded Church-Turing thesis. Another implication of our construction is that a Gaussian random Beltrami field on Euclidean space exhibits arbitrarily high computational complexity with probability 1. Finally, our proof also yields Turing complete flows and diffeomorphisms on $\mathbb{S}^2$ with zero topological entropy, thus disclosing a certain degree of independence within different hierarchies of complexity.

MSC:
35Q31 Euler equations
76B03 Existence, uniqueness, and regularity theory for incompressible inviscid fluids
68Q04 Classical models of computation (Turing machines, etc.)
68Q15 Complexity classes (hierarchies, relations among complexity classes, etc.)
35A10 Cauchy-Kovalevskaya theorems
35G60 Boundary value problems for systems of nonlinear higher-order PDEs
35A01 Existence problems for PDEs: global existence, local existence, non-existence
37B40 Topological entropy
58C40 Spectral theory; eigenvalue problems on manifolds

Keywords:
Euler equations; Beltrami fields; Turing machines; computational complexity

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References: