Kramer, Linus
Some remarks on proper actions, proper metric spaces, and buildings. (English)

This is a carefully written survey of properties of proper actions of topological groups on proper metric spaces, aimed at studying the actions of topological groups on buildings.

A map $f : X \to Y$ is called proper (following Bourbaki) if it is continuous and the pre-images of points are compact; in general topology, such maps are also known as perfect, and sometimes a proper map is defined, in a more general way, as one under which the pre-images of compact sets are compact. An action of a topological group $G$ on a topological space $X$ is proper if the associated map $G \times X \to X \times X$, sending a pair $(g, x)$ to $(gx, x)$, is proper. In particular, this definition implies that every point stabilizer $G_x$ is compact, and the quotient space $G/G_x$ is canonically homeomorphic to the orbit of $x$. A metric space $X$ is proper if the closed balls are compact (equivalently, a subset is compact if and only if it is closed and bounded). The isometry group $\text{Iso}(X)$ of a proper metric space $X$ is locally compact (and of course second-countable), and its canonical action on $X$ is proper. All those notions are the object of central interest for the article, yet in very concrete situations.

Section 3 of the article treats actions of topological groups by automorphisms of simplicial complexes and the conditions under which they are proper. The next class of examples of proper metric spaces, studied in Section 4, is given by simplicial complexes $\Delta$ equipped with the $M_\kappa$-structure, where $\kappa$ is a real number and $M_\kappa$ is a Riemannian manifold of constant sectional curvature $\kappa$, see [M. R. Bridson and A. Haefliger, Metric spaces of non-positive curvature. Berlin: Springer (1999; Zbl 0988.53001)]. For such spaces, it is determined when they are proper, and some properly acting groups of automorphisms are described. In Section 5, the author reviews buildings and groups properly acting on them, as well as various topologies on those groups and equivalent conditions for the actions being proper.

The paper contains a large number of illuminating examples as well as, importantly, counter-examples.

Reviewer: Vladimir Pestov (João Pessoa)

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